Dynamic Planar Range Maxima Queries

(presented at ICALP 2011)

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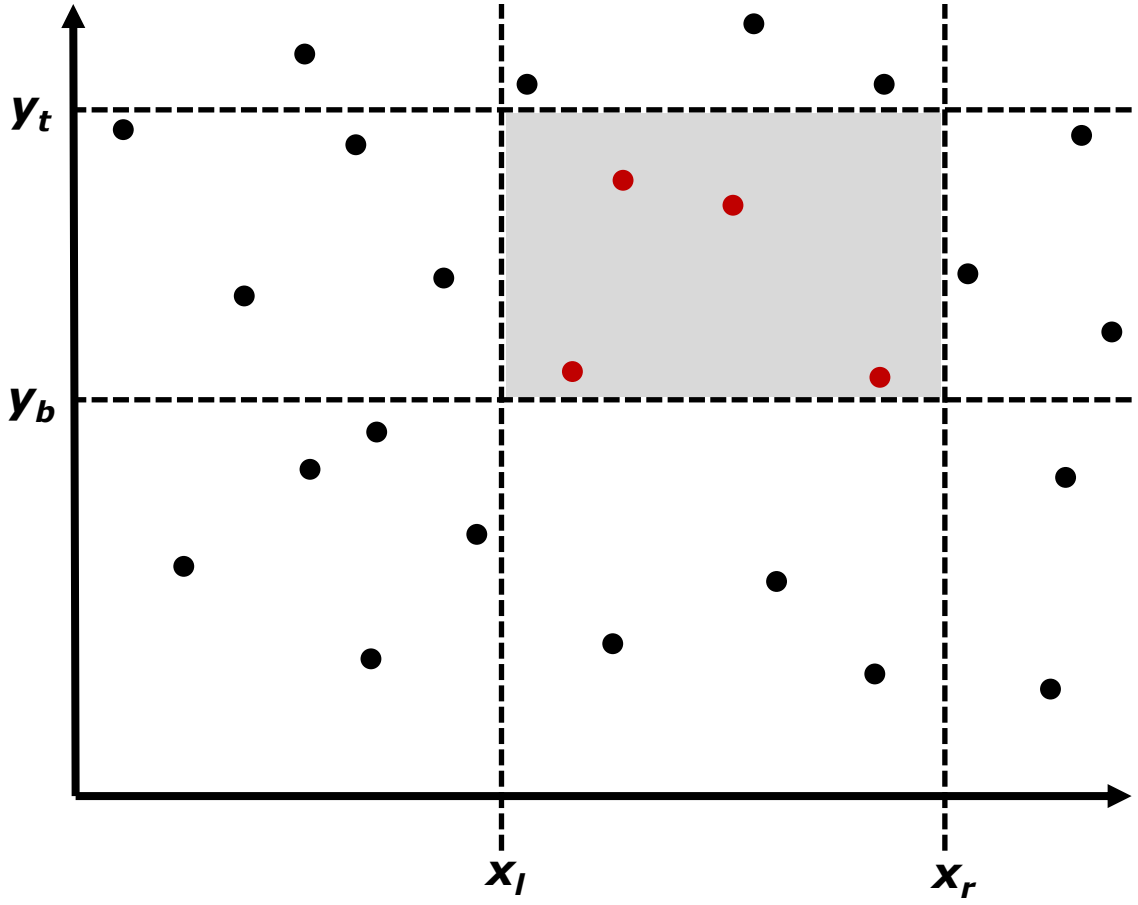
University of Primorska, October 10, 2011

Orthogonal Range Queries



Priority Search Tree [McCreight'75]

Space: O(*n*)

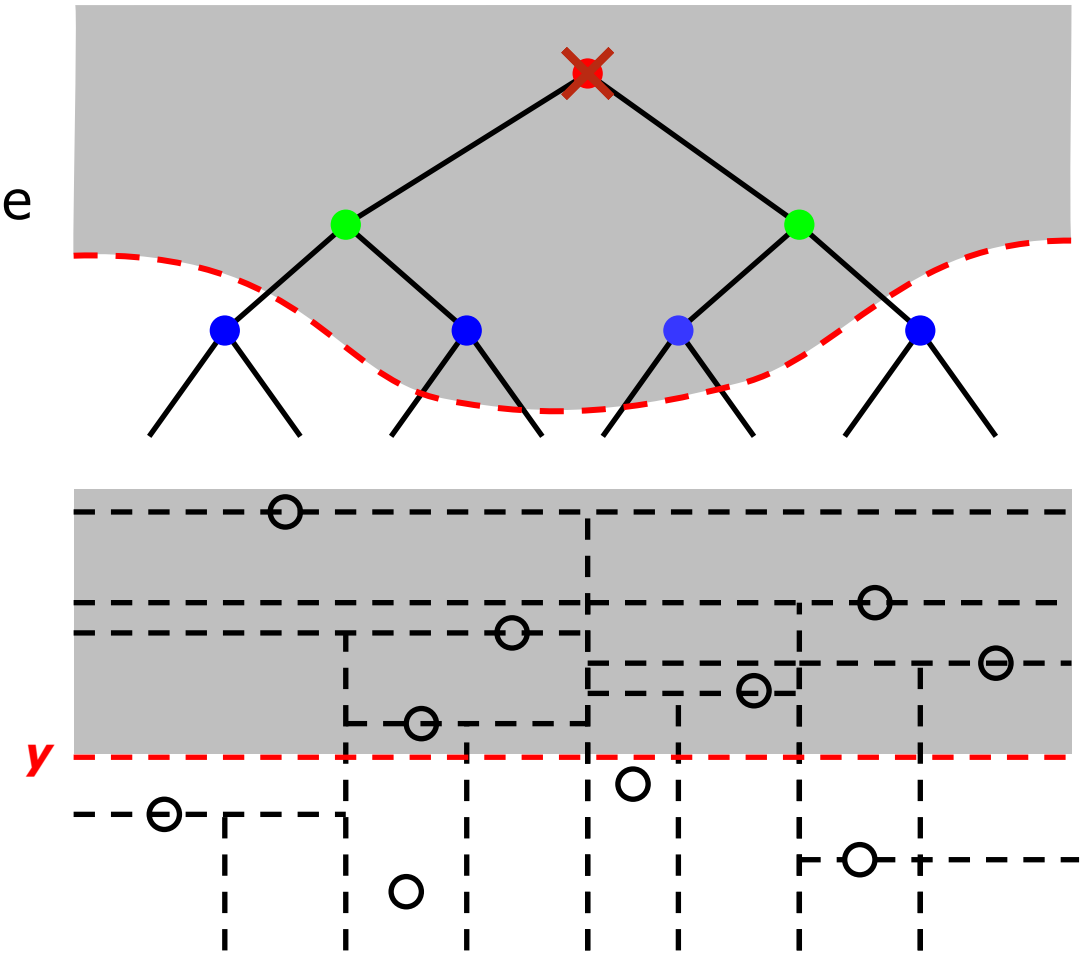
Update:

Recursively move up maximum *y*

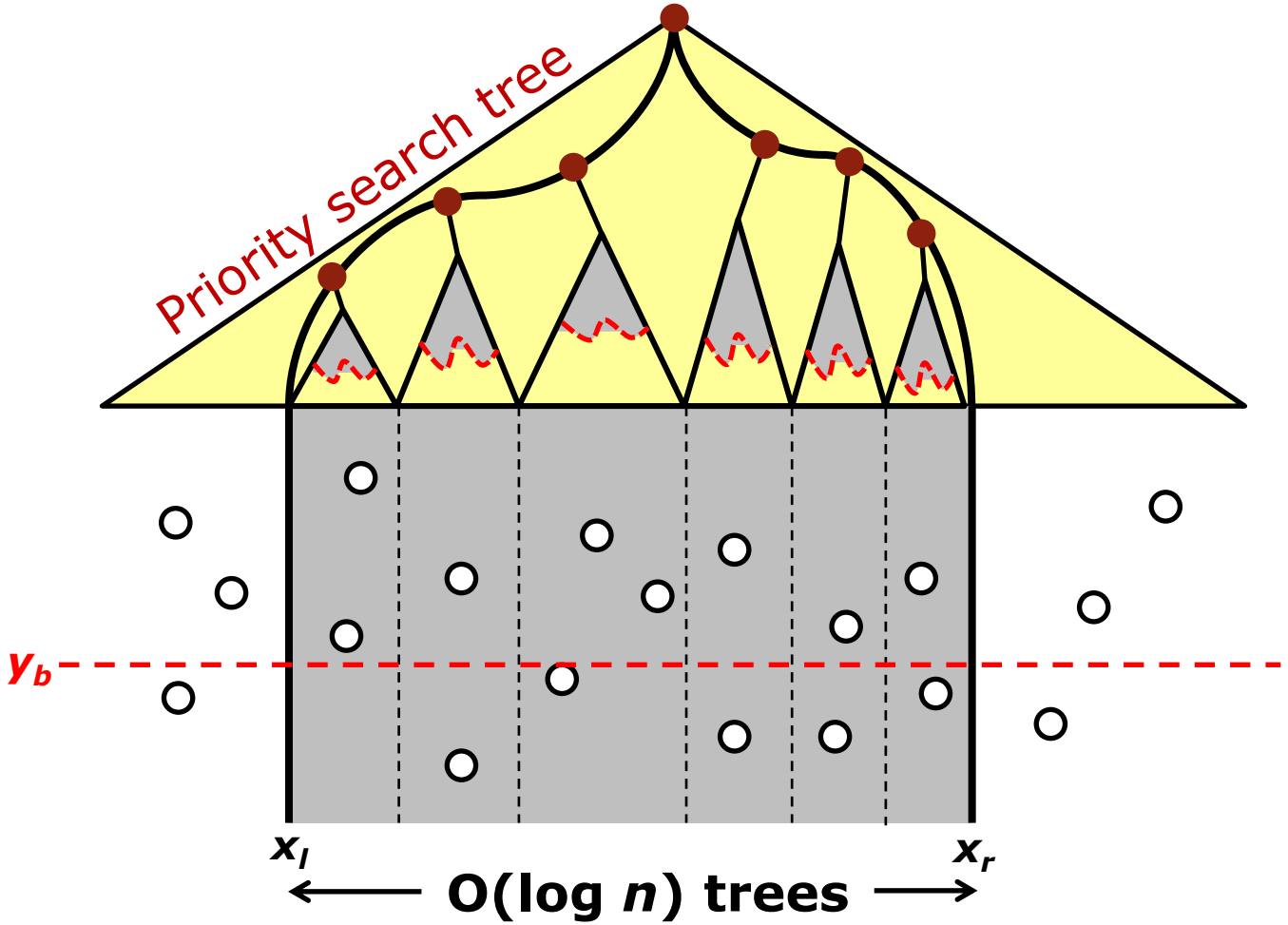
O(log *n*)

1-Sided reporting:

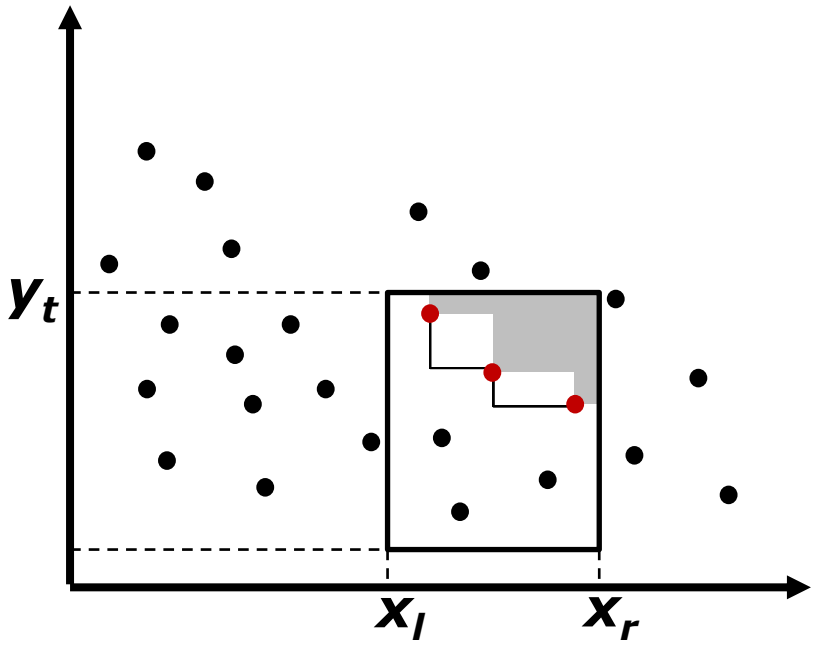
O(t)



3-Sided Reporting Queries O(log n + t)



Orthogonal Range MAXIMA Reporting alias "Generalized Planar SKYLINE Operator"



Dominance Maxima Queries

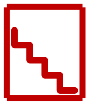
Report **all maximal points** among points with x in $[x_l,+\infty)$ and y in $[y_b,+\infty)$

Contour Maxima Queries

Report **all maximal points** among points with x in $(-\infty, x_r]$

3-Sided Maxima Queries

Report **all maximal points** among points with x in $[x_l, x_r]$ and y in $[y_b,+\infty)$



4-Sided Maxima Queries

Report **all maximal points** among points with x in $[x_l, x_r]$ and y in $[y_b,y_t]$

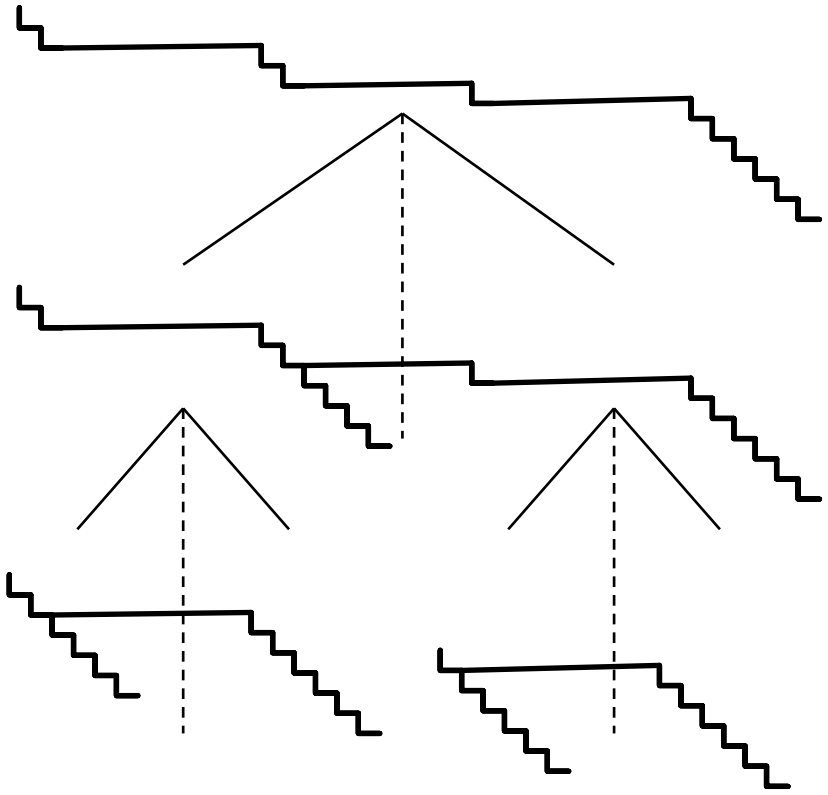
Static maximal points in O(n·log n) time [Kung, Luccio, Preparata, J.ACM'75]

Dynamic Range Maxima Reporting

	Space			Insert	Delete
Overmars, van Leeuwen '81	n	$\log n + t$	$\log^2 n + t$	$\log^2 n$	
Frederickson, Rodger '90	n	$\log n + t$	$\log^2 n + t$ $\log n(1+t)$	$\log n$	$\log^2 n$
Janardan '91	n	$\log n + t$		$\log n$	$\log^2 n$
Kapoor '00	n	$\log n + t$ amo.	-	$\log n$	
[ICALP '11] 	n	$\log n$ / loglog $n + t$		$\log n$ / loglog n	
	$n \cdot \log n$	$\log^2 n$ / loglog $n + t$		$\log^2 n$ / loglog n	

RAM

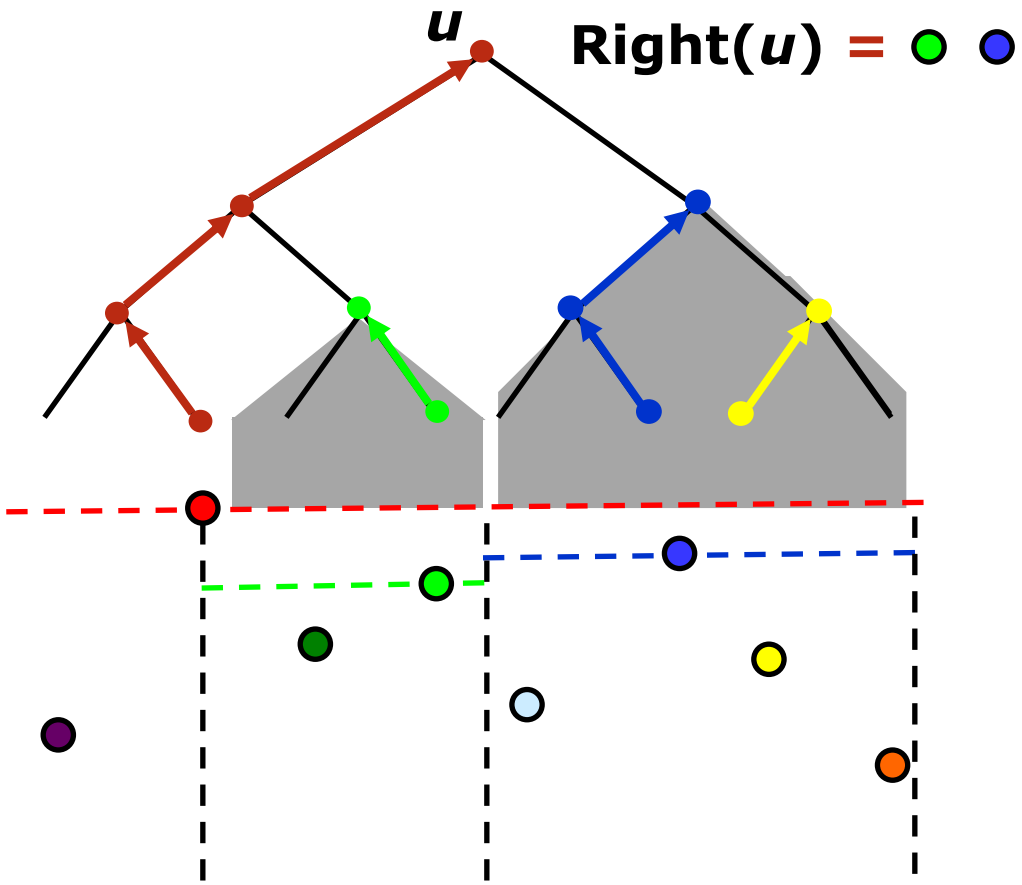
Overmars, van Leeuwen [JCSS '81]



O(log n + t)

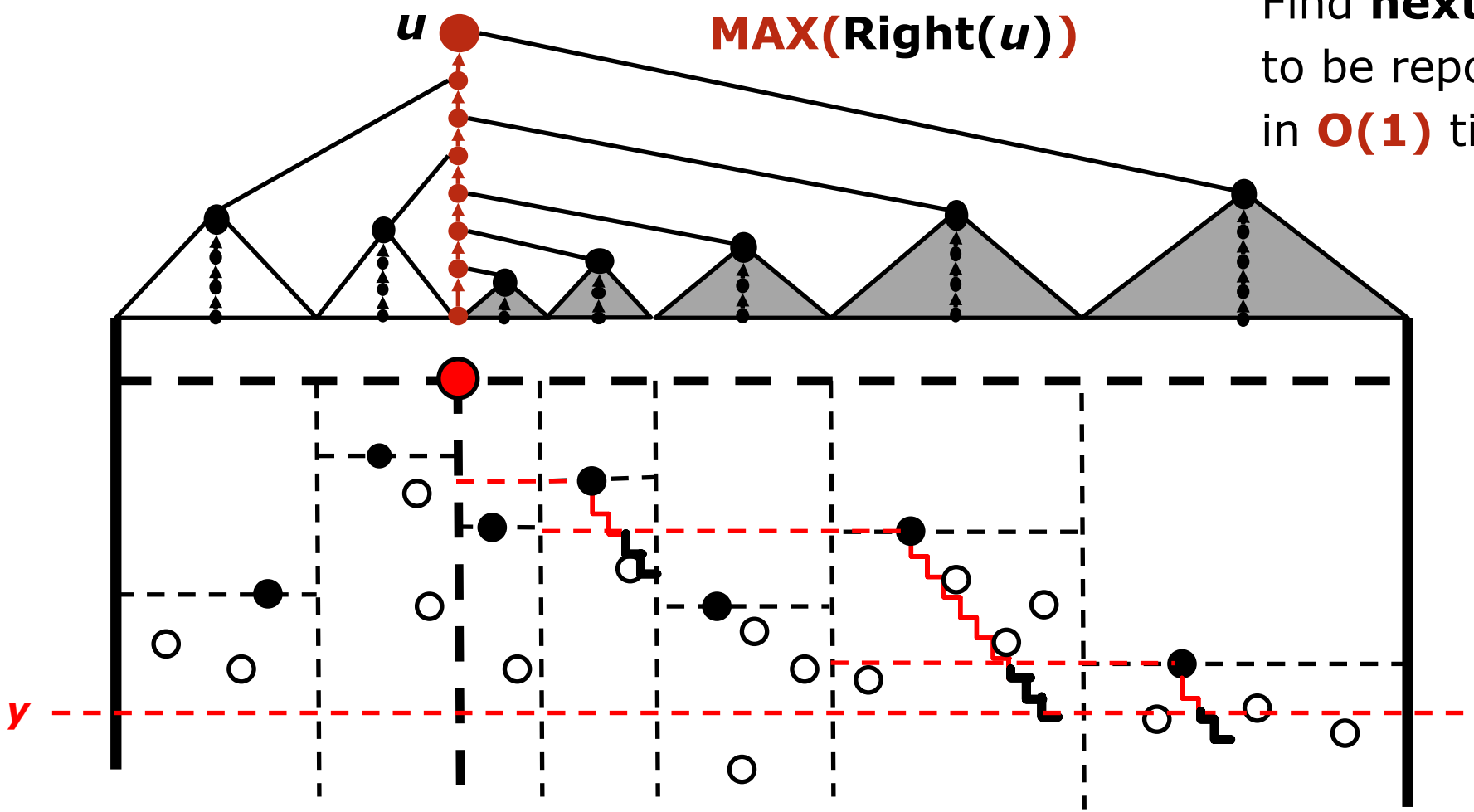
Updates: O(log^2 n)

Our Structure - Tournament Tree



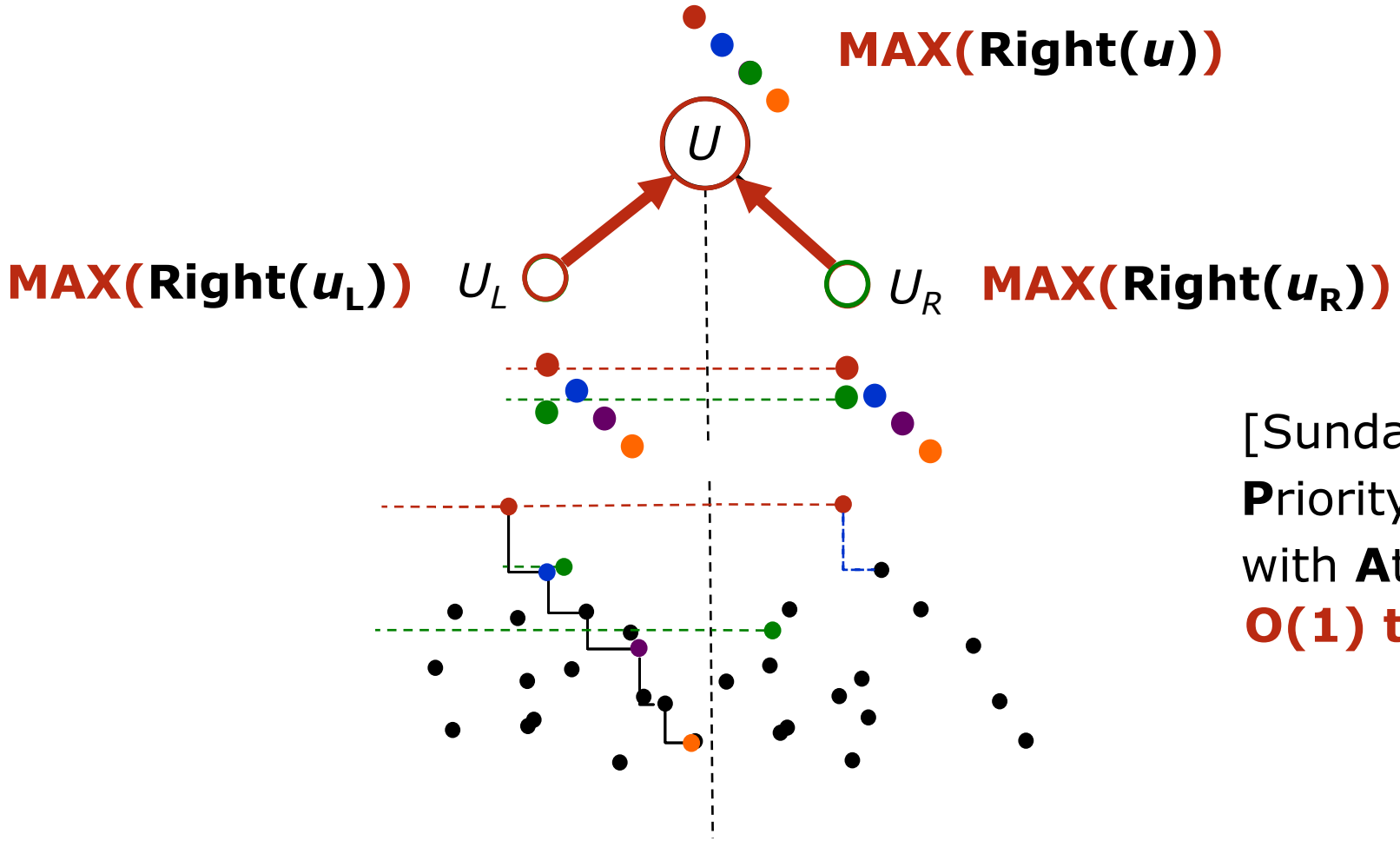
Copy Up
Maximum *y*

Tournament Tree



Find **next** point to be reported in **O(1)** time

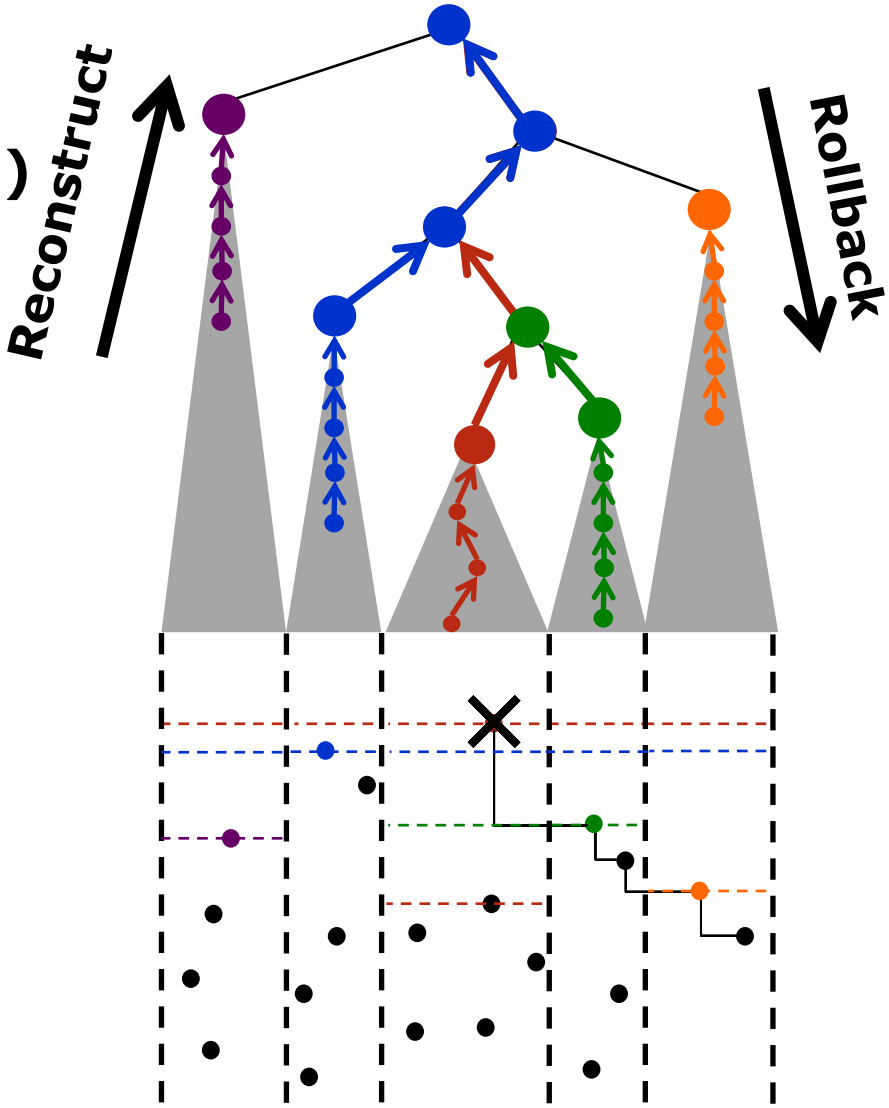
Computation of MAX(Right(*u*))



[Sundar '89]
Priority **Q**ueue with **A**ttrition
O(1) time

Update Operation

Space:**O(*n*)**
Update:**O(log *n*)**



Partially **P**ersistent **P**riority **Q**ueue with **A**ttrition

[Driscol et al. '89]

O(1) **amortized** time, space overhead per update step

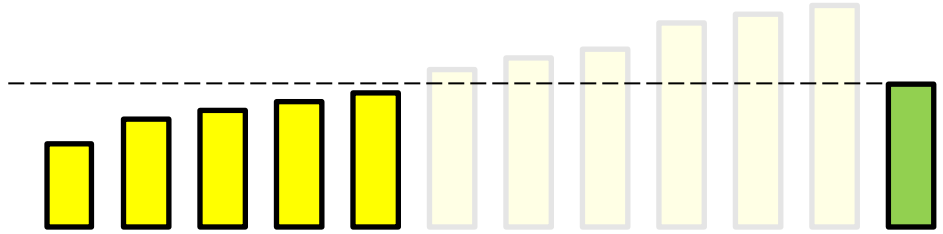
[Brodal '96]

worst case

Priority Queues with Attrition

[Sundar, IPL '89]

- Deletemin()
- InsertAndAttrite(element)

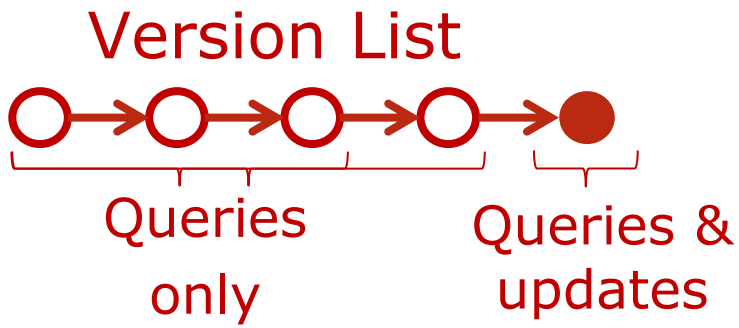


O(1) worst case time

Partial Persistent Data Structures

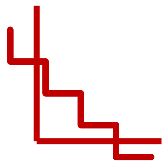
[Driscoll et al., JCSS ’89]

- “Persistent” = remember previous versions
- Any pointer-based structure with O(1) indegree

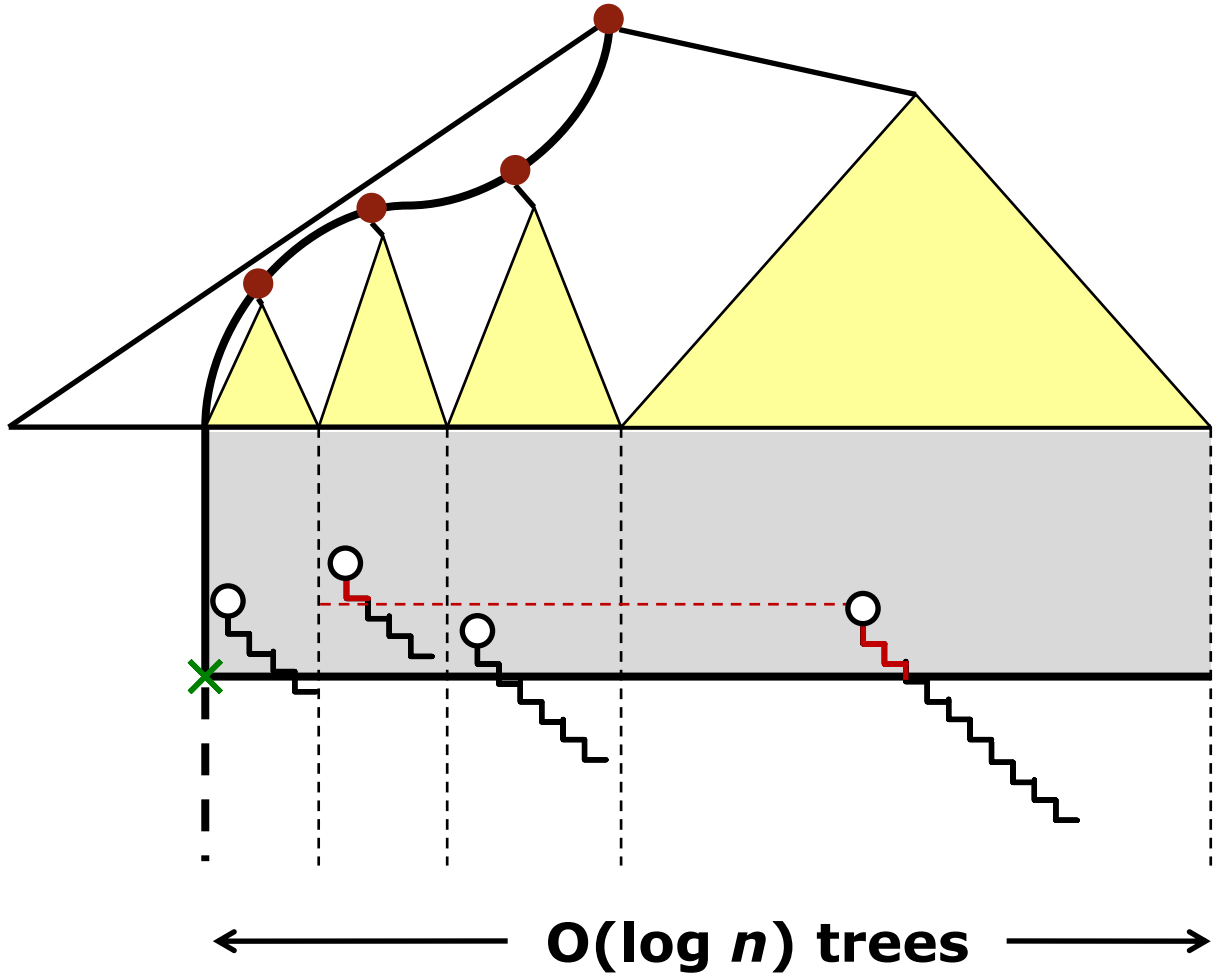


[Brodal, NJC ’96]

- O(1) worst case time overhead per access step
- O(1) **worst case** time, space overhead per update step
- “Rollback” = discard latest version in O(Update) time

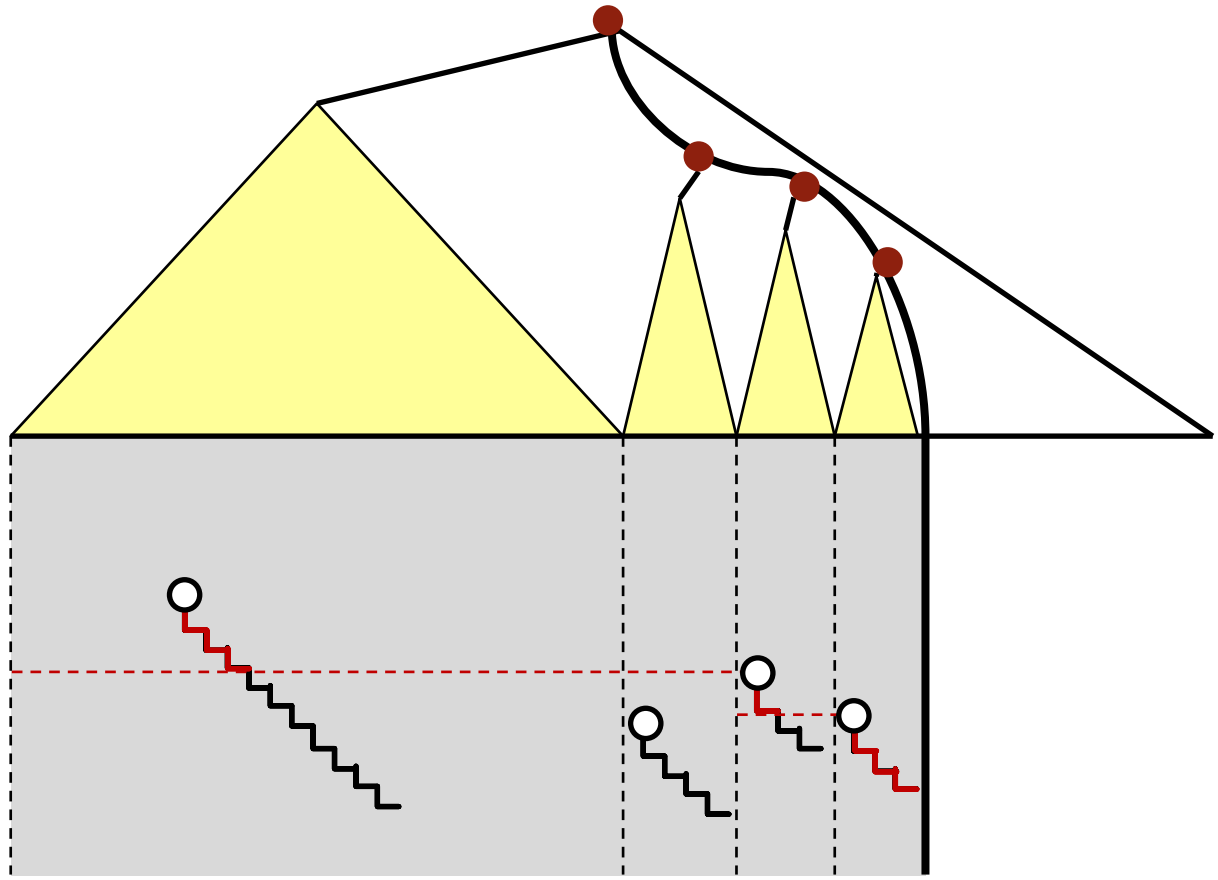


Dominance Range Maxima Queries



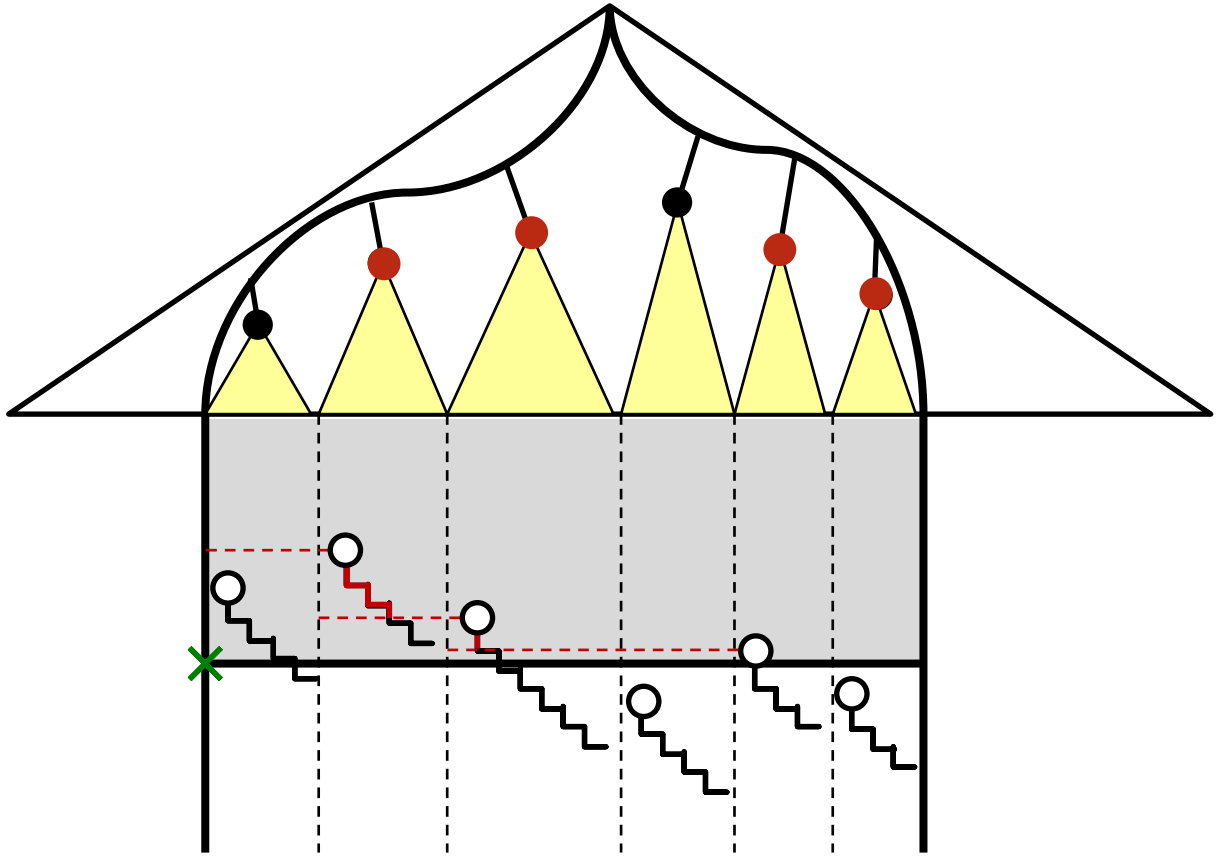
Query time O(log n + t)

Contour Range Maxima Queries



Query time O(log n + t)

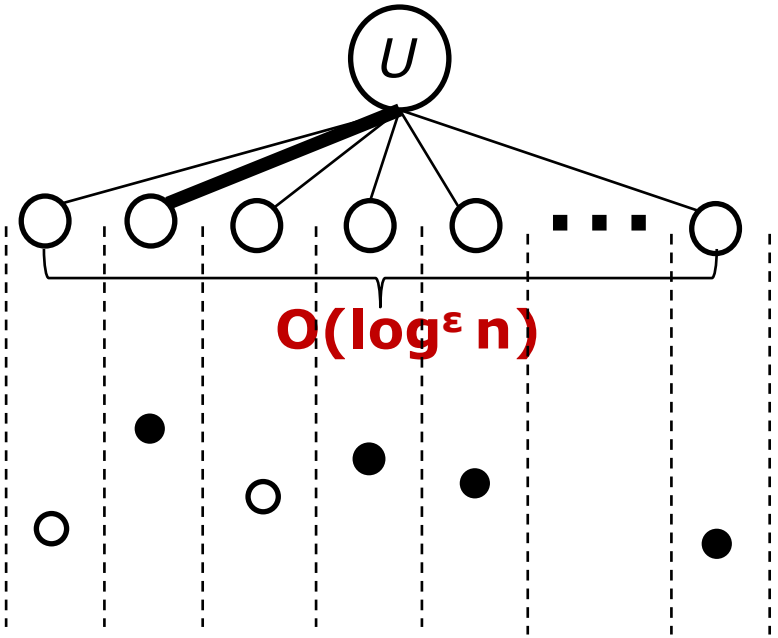
3-Sided Range Maxima Queries



← O(log *n*) trees →

Query time O(log *n* + *t*)

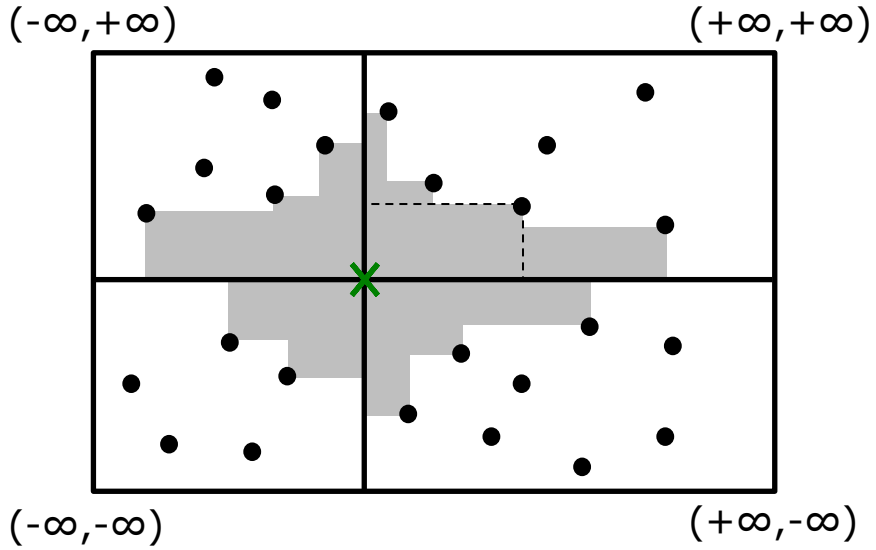
RAM – O(log *n*/loglog *n* + *t*)

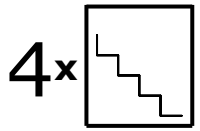


- Height O(log *n* / loglog *n*)
- MAX(Right(u)) maintained using **Q-heaps** [Fredman, Willard, JCSS ´94]

4-Sided Range MAXIMA Reporting and Rectangular Visibility Queries

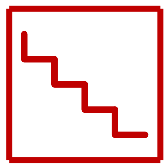
Proximity Queries/Similarity Search



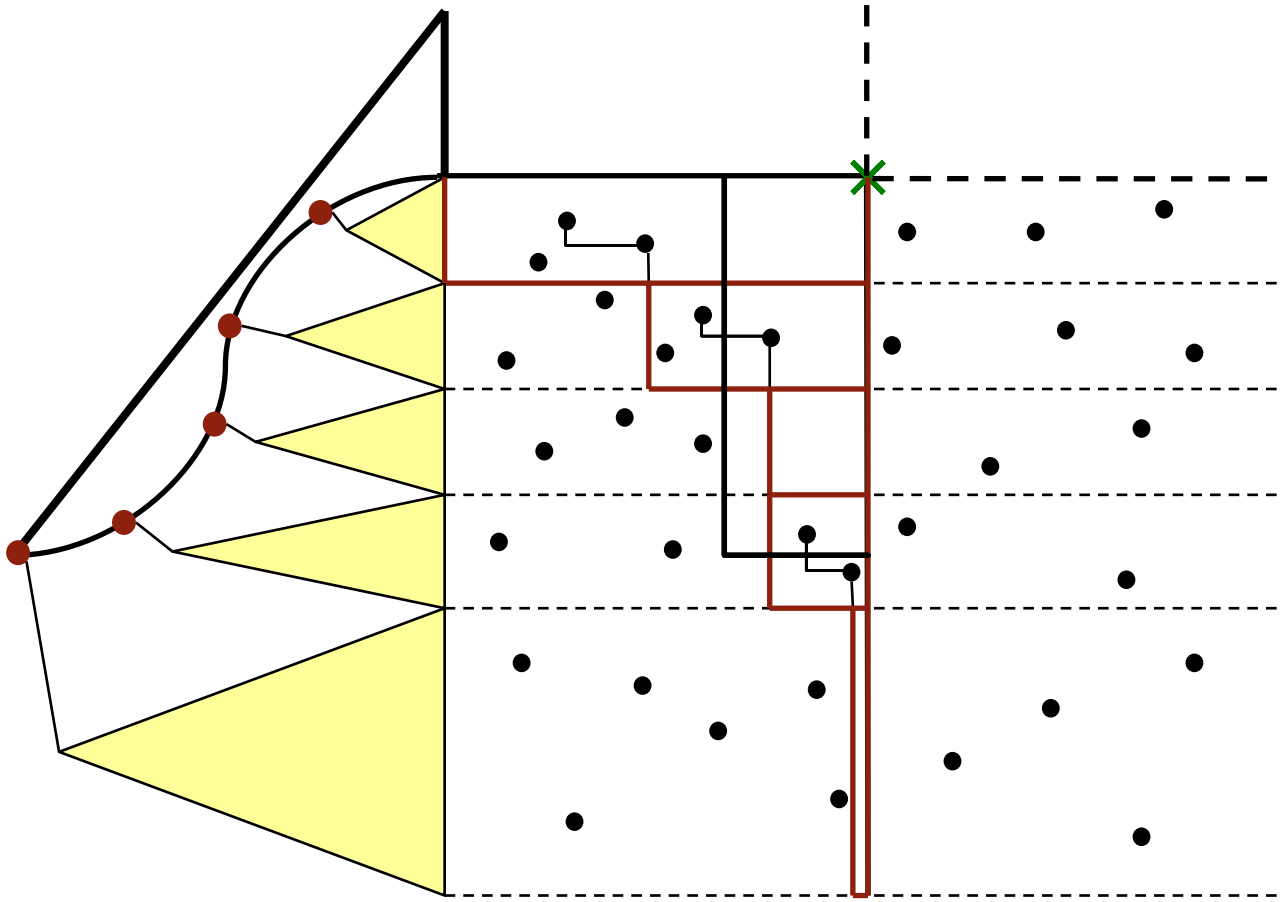


4-Sided Range Maxima Queries

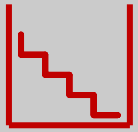
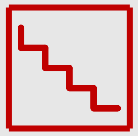
	Space	✿	Insert	Delete
Overmars, Wood '88	$n \cdot \log n$	$\log^2 n + t$ $\log^2 n + t \cdot \mathbf{\log n}$	$\log^2 n$	$\mathbf{\log^3 n}$ $\log^2 n$
[ICALP '11]	$\boldsymbol{n \cdot \log n}$	$\boldsymbol{\log^2 n + t}$	$\boldsymbol{\log^2 n}$	$\boldsymbol{\log^2 n}$



4-sided Range Maxima Queries



Query time O(log^2 n + t), space O(n·log n)

	Space	Query	Insert/Delete	
	$O(n)$	$O(\log n/\log\log n + t)$	$O(\log n/\log\log n)$	RAM
	$O(n \cdot \log n)$	$O(\log^2 n + t)$	$O(\log^2 n/\log\log n)$	

Thank You

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