PRIME -TIME

GERTH STØLTING BRODAL, VERDENS KEDELIGSTE FOREDRAG 4/3-1

DEFINITION



DEFINITION

PRIMTAL $b \geq 2$ ER EL HELTAL b TE

HVIS OG KUN HVIS $1 \text{ OG } p$ ER DE ENESTE

POSITIVE HELTAL DER GÅR OP I p.

DEFINITION

Et heltal $p \geq 2$ er et **primtal**, hvis og kun hvis 1 og p er de eneste positive heltal der går op i p.

EKSEMPEL

$$2035015925856908896847842181143$$

IKKE ET PRIMTAL

DEFINITION

ET HELTAL $p \geq 2$ ER ET PRIMTAL

HVIS OG KUN HVIS 1 OG p ER DE ENESTE

POSITIVE HELTAL DER GÅR OP I p.

EKSEMPEL

$$20350159258569088968478421818143$$

IKKE ET PRIMTAL

$$= 26347623547272733 * 772371717777771$$

41

ER ET PRIMTAL

BEVIS

1		41		0		
2		20		1		
3		13		2		
4		10		1		
5		8		1		
6		6		5		
7		5		6		
8	*	5	+	1	=	41
9		4		5		
10		4		1		
11		3		8		
12		3		5		
13		3		2		
14		2		13		
15		2		11		
16		2		9		
17		2		7		
18		2		5		
19		2		3		

41

ER ET PRIMTAL

BEVIS

13	3	2	
14	2	13	
15	2	11	
16	2	9	
17	2	7	
18	2	5	
19	2	3	
20	2	1	
21	1	20	
22	1	19	
23	1	18	
24	1	17	
25	1	16	
26	1	15	
27	1	14	
28	1	13	
29	1	12	
30	1	11	
31	1	10	
32	1	9	
33	1	8	

* + = 41

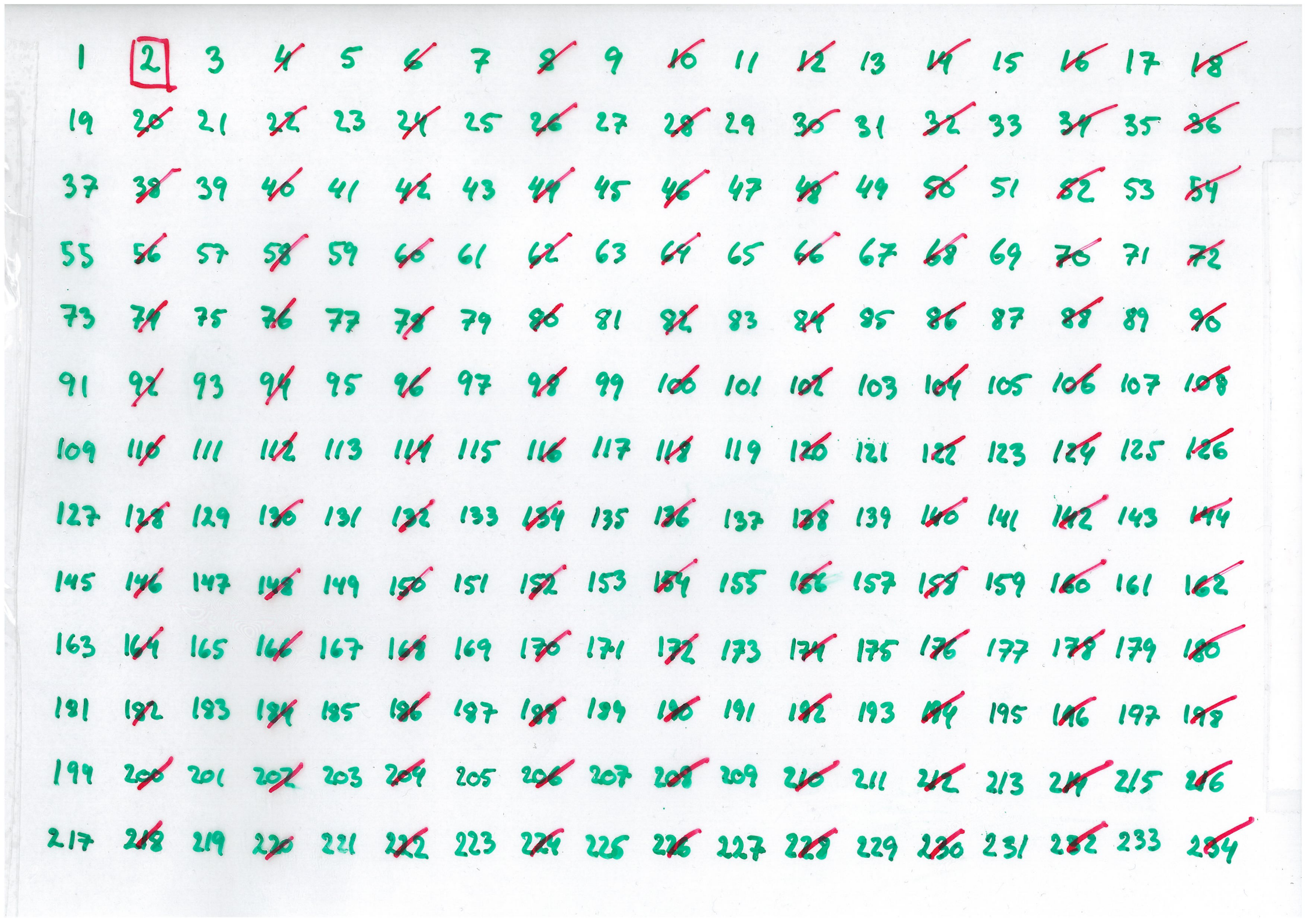
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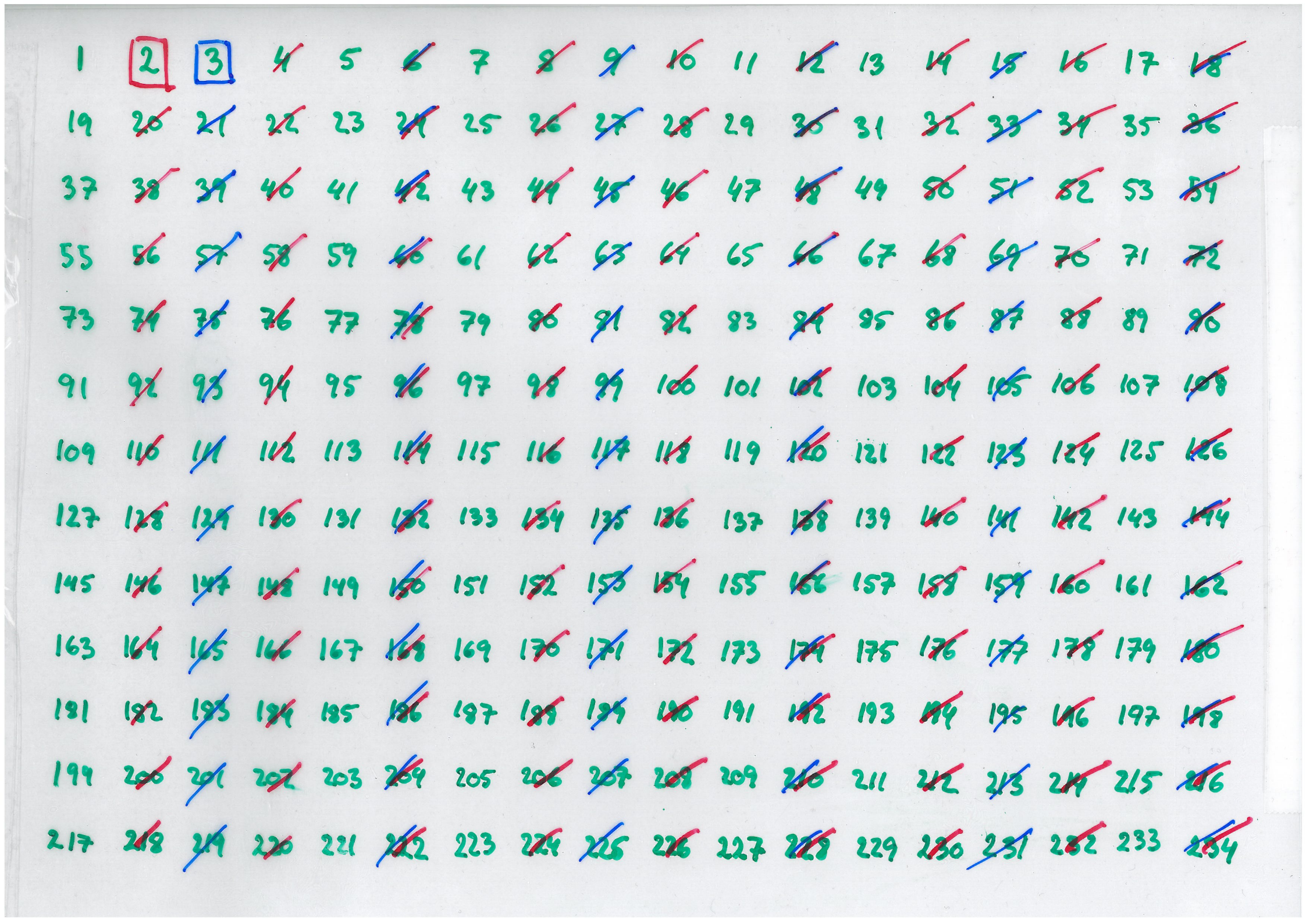
ER ET PRIMTAL

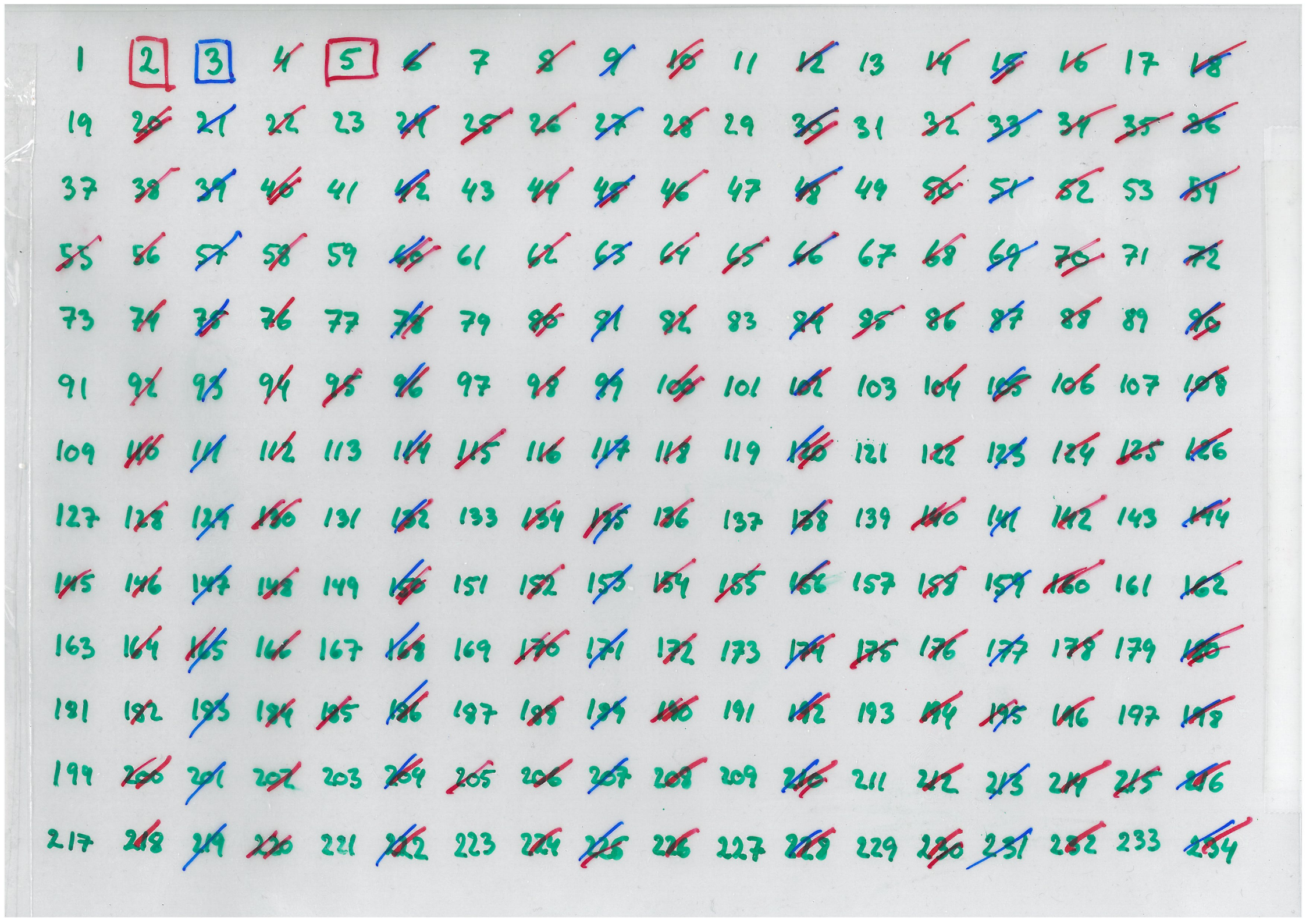
BEVIS

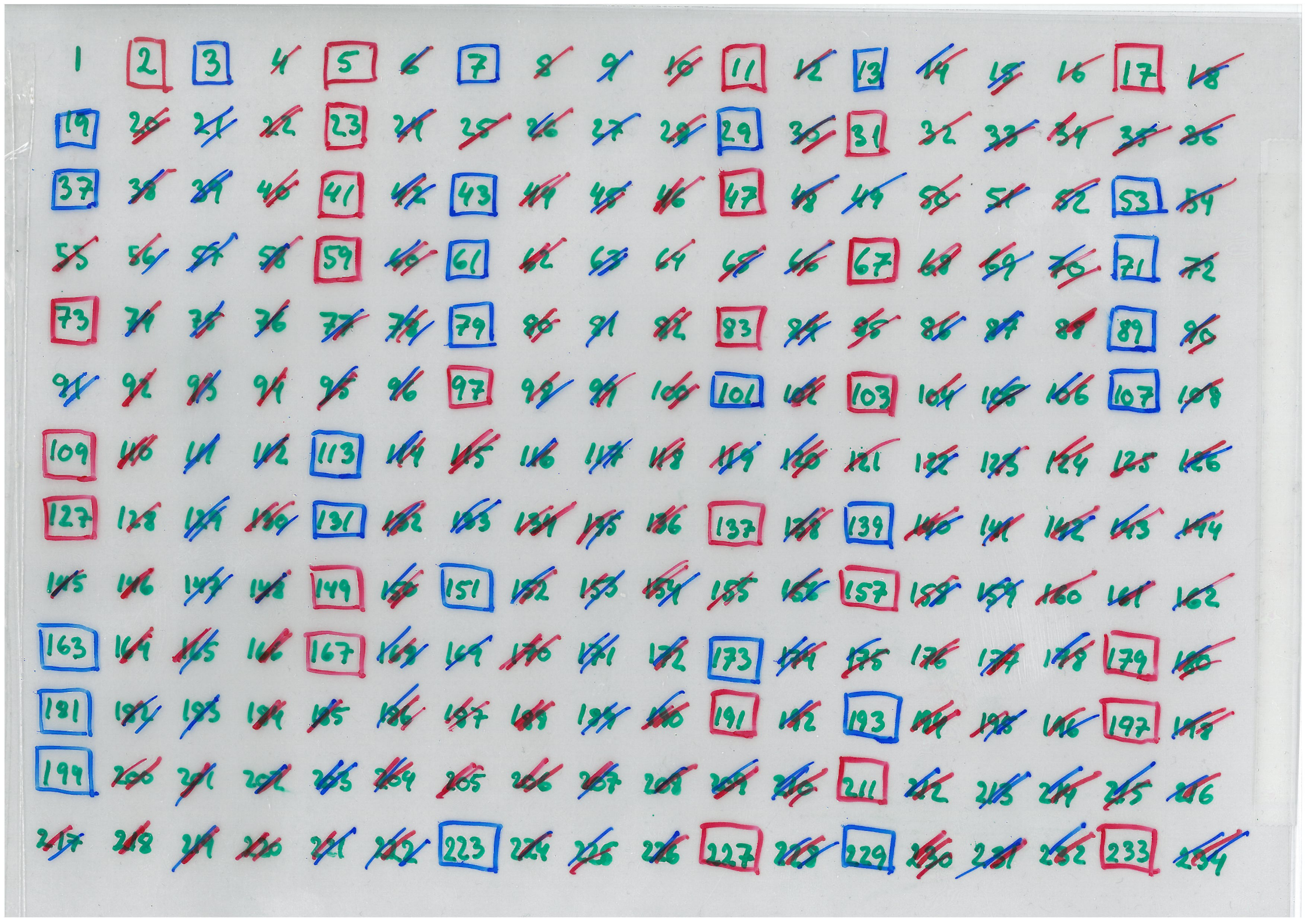
22	1		19
23	1		18
24	1		17
25	1		16
26	1		15
27	1		14
28	1		13
29	1		12
30	1		11
31	1		10
32*	1	+	9 = 41
33	1		8
34	1		7
35	1		6
36	1		5
37	1		4
38	1		3
39	1		2
40	1		1
41	1		0

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54
55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72
73	74	75	76	77	78	79	80	81	82	83	84	95	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108
109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126
127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144
145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162
163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180
191	182	183	184	185	186	187	189	189	190	191	192	193	194	195	196	197	198
199	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216
217	218	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234











ERATOSTHENES

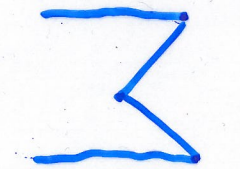
276 - 194 BC

ERATOSTHENES' SI

— ANALYSE —

ERATOSTHENES' SI

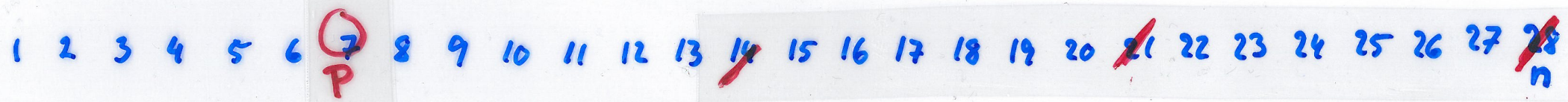
— ANALYSE —



ERATOSTHENES' SI

— ANALYSE —

$$\sum_{\text{PRIMTAL } p \leq n} \left\lfloor \frac{n}{p} \right\rfloor - 1$$



ERATOSTHENES' SI

— ANALYSE —

$$\sum_{\substack{\text{PRIMTAL} \\ p \leq n}} \left\lfloor \frac{n}{p} \right\rfloor - 1 \leq n \cdot \sum_{\substack{\text{PRIMTAL} \\ p \leq n}} \frac{1}{p}$$

1 2 3 4 5 6 (7) 8 9 10 11 12 13 ~~14~~ 15 16 17 18 19 20 ~~21~~ 22 23 24 25 26 27 ~~28~~

p n

ERATOSTHENES' SI

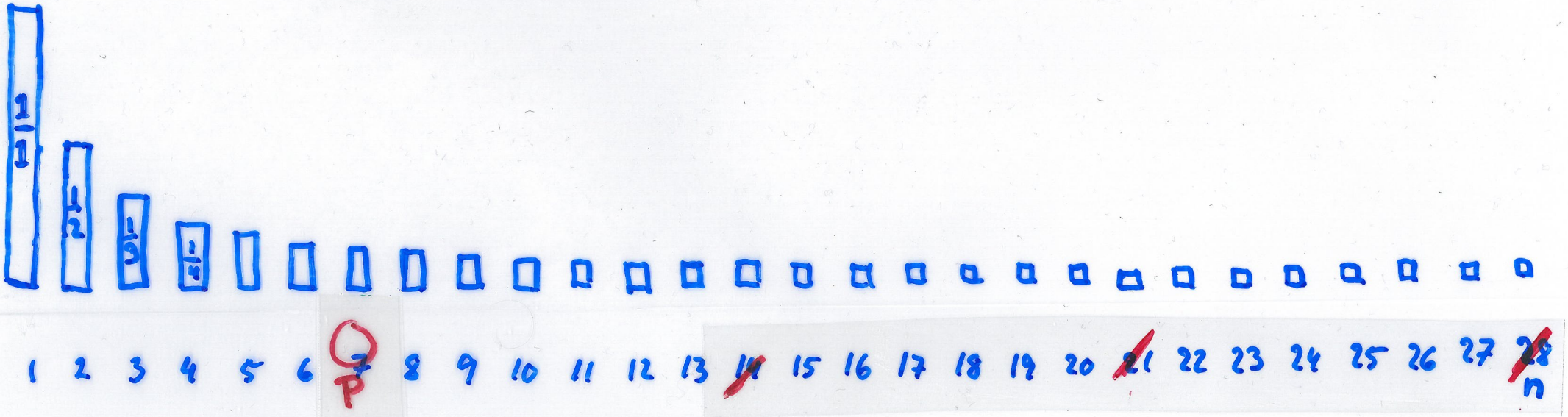
— ANALYSE —

$$\sum_{\substack{\text{PRIMTAL} \\ p \leq n}} \left\lfloor \frac{n}{p} \right\rfloor - 1 \leq n \cdot \sum_{\substack{\text{PRIMTAL} \\ p \leq n}} \frac{1}{p} \leq n \cdot \sum_{1 \leq i \leq n} \frac{1}{i}$$



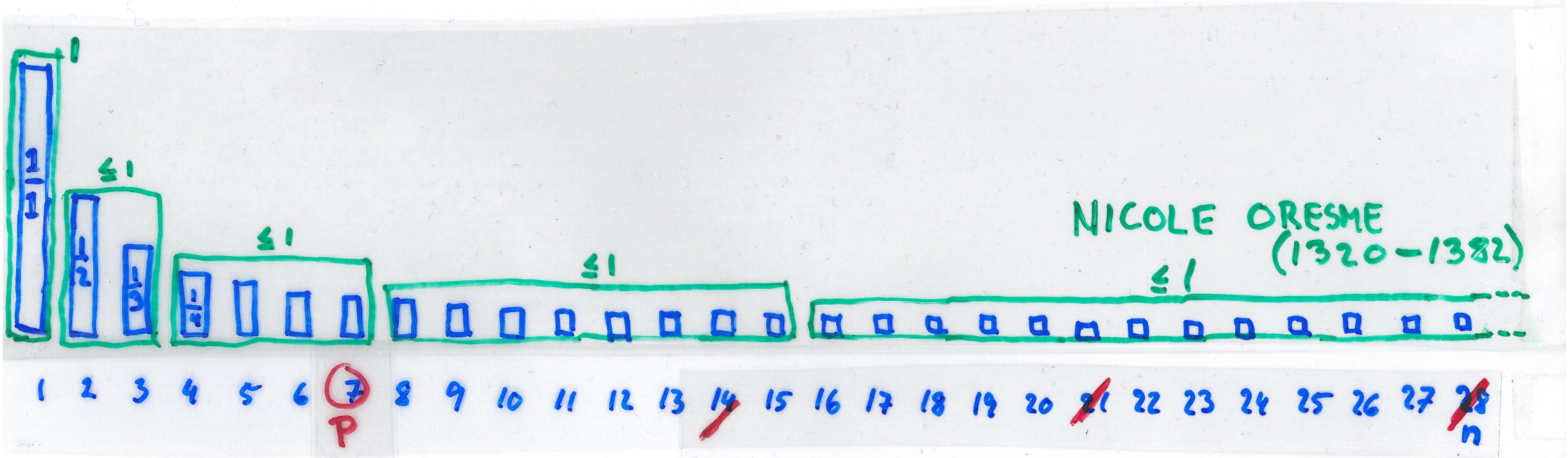
ERATOSTHENES' SI
— ANALYSE —

$$\sum_{\substack{\text{PRIMTAL} \\ p \leq n}} \left(\left\lfloor \frac{n}{p} \right\rfloor - 1 \right) \leq n \cdot \sum_{\substack{\text{PRIMTAL} \\ p \leq n}} \frac{1}{p} \leq n \cdot \sum_{1 \leq i \leq n} \frac{1}{i}$$



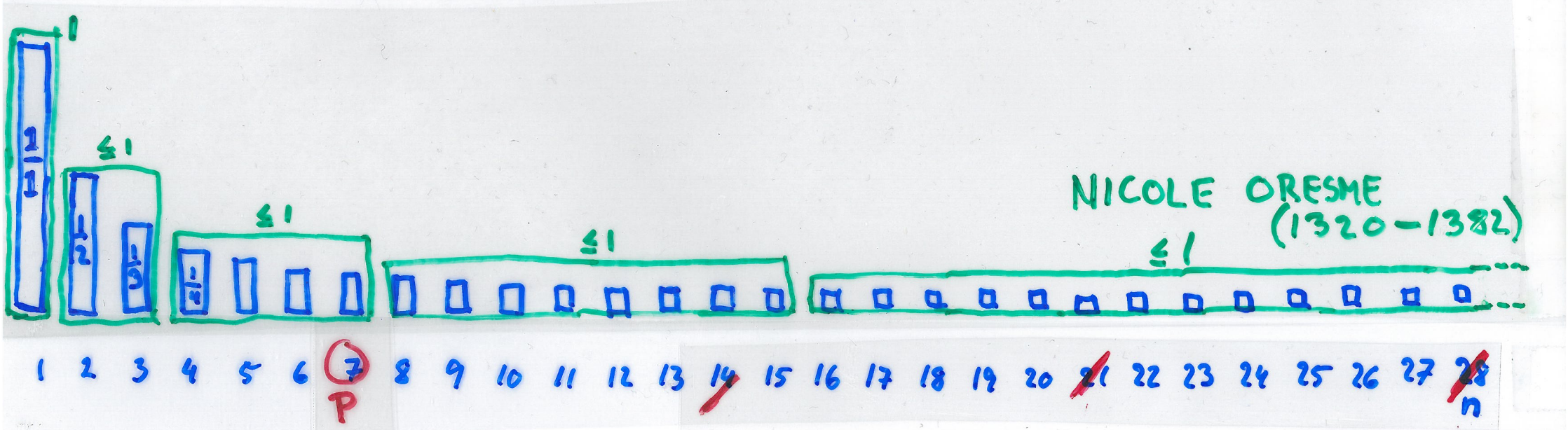
ERATOSTHENES' SI
— ANALYSE —

$$\sum_{\substack{\text{PRIMTAL} \\ p \le n}} \left(\left\lfloor \frac{n}{p} \right\rfloor - 1 \right) \le n \cdot \sum_{\substack{\text{PRIMTAL} \\ p \le n}} \frac{1}{p} \le n \cdot \sum_{1 \le i \le n} \frac{1}{i}$$



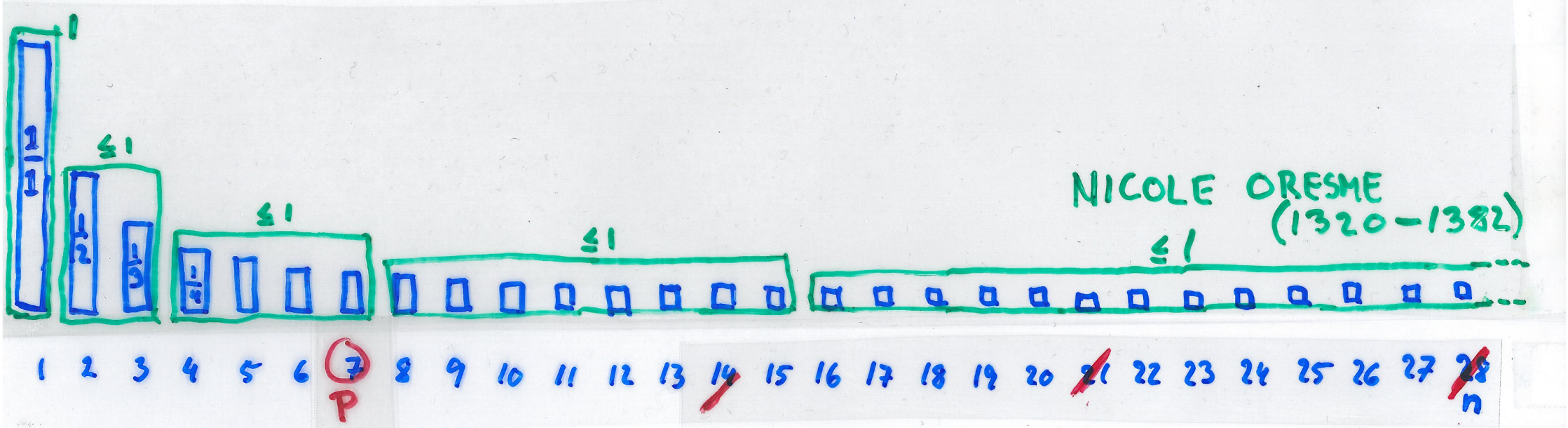
ERATOSTHENES' SI
— ANALYSE —

$$\sum_{\substack{\text{PRIMTAL} \\ p \leq n}} \left\lfloor \frac{n}{p} \right\rfloor - 1 \leq n \cdot \sum_{\substack{\text{PRIMTAL} \\ p \leq n}} \frac{1}{p} \leq n \cdot \sum_{1 \leq i \leq n} \frac{1}{i} \leq n \cdot (\log_2 n + 1)$$



ERATOSTHENES' SI
— ANALYSE —

$$\sum_{\substack{\text{PRIMTAL} \\ p \leq n}} \left\lfloor \frac{n}{p} \right\rfloor - 1 \leq n \cdot \sum_{\substack{\text{PRIMTAL} \\ p \leq n}} \frac{1}{p} \leq 3 + 2n \cdot \log_2 \log_2 n$$



DEFINITION

$\sum \frac{1}{p} \leq$

PRIMTAL

$p \leq n$

DEFINITION

$$\sum \frac{1}{p} \leq$$

PRIMTAL

$p \leq n$

DEFINITION

$\pi(n)$ = "ANTAL PRIMTAL $\leq n$"

$\pi(234) = 51$

$\leq$ $\leq$ $=$ $\leq$

$\Rightarrow$ $\leq$

$\sum_{\text{PRIMTAL } p \leq n} \frac{1}{p} \leq$ $\leq$ $\leq$

$\leq$ $\leq$ $\square$

$\underset{2^1}{2}$ 3 | 5 7 | 11 13 | 17 19 23 29 31 | 37 41 43 47 53 59 61 | 67 71 73 79

$4 = 2^2$ $8 = 2^3$ $16 = 2^4$ $32 = 2^5$ n $64 = 2^6$

DEFINITION

$\pi(n) =$ "ANTAL PRIMTAL $\leq n$"

$\pi(234) = 51$

$$\leq \frac{2n \cdot (2n-1) \cdot (2n-2) \cdots (n+2) \cdot (n+1)}{n \cdot (n-1) \cdot (n-2) \cdots 2 \cdot 1} = \leq$$

$\leq$

$\Rightarrow$

$\leq$

$\sum_{\substack{p \text{ PRIMTAL} \\ p \leq n}} \frac{1}{p} \leq$

$\leq$

$\leq$

$\leq$

$\square$

2 3 5 7 11 13 17 19 23 29 31 37 41 43 47 53 59 61 67 71 73 79

2^1 $4=2^2$ $8=2^3$ $16=2^4$ $32=2^5$ n $64=2^6$

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$\leq$

$\Rightarrow$

$\leq$

$\sum_{\text{PRIMTAL } p \leq n} \frac{1}{p} \leq$

$\leq$

$\leq$

$\leq$

$\square$

2 | 3 | 5 | 7 | 11 | 13 | 17 | 19 | 23 | 29 | 31 | 37 | 41 | 43 | 47 | 53 | 59 | 61 | 67 | 71 | 73 | 79

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$\leq$

$\Rightarrow$

$\leq$

$\sum_{\substack{p \text{ PRIMTAL} \\ p \leq n}} \frac{1}{p} \leq$

$\leq$

$\leq$

$\leq$

$\square$

2 | 3 | 5 | 7 | 11 | 13 | 17 | 19 | 23 | 29 | 31 | 37 | 41 | 43 | 47 | 53 | 59 | 61 | 67 | 71 | 73 | 79

2^1 $4=2^2$ $8=2^3$ $16=2^4$ $32=2^5$ n $64=2^6$

ALLE PRIMTAL $n < p \leq 2n$ DIVIDERER $\binom{2n}{n}$

DEFINITION

$\pi(n) =$ "ANTAL PRIMTAL $\leq n$"

$\pi(234) = 51$

$$\prod_{\text{PRIMTAL } p,\ n<p\leq 2n} p \leq \frac{2n\cdot(2n-1)\cdot(2n-2)\cdots(n+2)\cdot(n+1)}{n\cdot(n-1)\cdot(n-2)\cdots 2\cdot 1} = \binom{2n}{n} \leq 2^{2n}$$

$\Rightarrow$

$$\sum_{\text{PRIMTAL } p\leq n} \frac{1}{p} \leq \quad \leq \quad \leq \quad \leq \quad \square$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79
2^1	2	3	$4=2^2$	5	6	7	$8=2^3$	9	10	11	12	13	14	15	$16=2^4$	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	$32=2^5$	33	34	35	36	37	38	39	40	41	42	43	44	45	46	n	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	$64=2^6$	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79

ALLE PRIMTAL $n < p \leq 2n$ DIVIDERER $\binom{2n}{n}$

DEFINITION

$\pi(n) =$ "ANTAL PRIMTAL $\leq n$"

$\pi(234) = 51$

$$n^{\pi(2n)-\pi(n)} \leq \prod_{\substack{\text{PRIMTAL } p \\ n<p\leq 2n}} p \leq \frac{2n\cdot(2n-1)\cdot(2n-2)\cdots(n+2)\cdot(n+1)}{n\cdot(n-1)\cdot(n-2)\cdots 2\cdot 1} = \binom{2n}{n} \leq 2^{2n}$$

$\Rightarrow$

$$\sum_{\substack{\text{PRIMTAL} \\ p\leq n}} \frac{1}{p} \leq \quad \leq \quad \leq \quad \leq \quad \square$$

2 | 3 | 5 | 7 | 11 | 13 | 17 | 19 | 23 | 29 | 31 | 37 | 41 | 43 | 47 | 53 | 59 | 61 | 67 | 71 | 73 | 79

2^1 $4=2^2$ $8=2^3$ $16=2^4$ $32=2^5$ n $64=2^6$

ALLE PRIMTAL DIVIDERER $\binom{2n}{n}$

DEFINITION

$\pi(n) =$ "ANTAL PRIMTAL $\leq n$"

$\pi(234) = 51$

$n < p \leq 2n$

$$n^{\pi(2n)-\pi(n)} \leq \prod_{\substack{\text{PRIMTAL } p \\ n<p\leq 2n}} p \leq \frac{2n\cdot(2n-1)\cdot(2n-2)\cdots(n+2)\cdot(n+1)}{n\cdot(n-1)\cdot(n-2)\cdots 2\cdot 1} = \binom{2n}{n} \leq 2^{2n}$$

$$\Rightarrow \quad \pi(2n)-\pi(n) \leq \frac{2n}{\log_2 n}$$

$$\sum_{\substack{\text{PRIMTAL} \\ p\leq n}} \frac{1}{p} \leq \quad \leq \quad \leq \quad \leq \quad \square$$

2 | 3 | 5 | 7 | 11 | 13 | 17 | 19 | 23 | 29 | 31 | 37 | 41 | 43 | 47 | 53 | 59 | 61 | 67 | 71 | 73 | 79

2^1 $4=2^2$ $8=2^3$ $16=2^4$ $32=2^5$ n $64=2^6$

ALLE PRIMTAL $n < p \leq 2n$ DIVIDERER $\binom{2n}{n}$

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$$n^{\pi(2n)-\pi(n)} \leq \prod_{\substack{\text{PRIMTAL } p \\ n<p\leq 2n}} p \leq \frac{2n\cdot(2n-1)\cdot(2n-2)\cdots(n+2)\cdot(n+1)}{n\cdot(n-1)\cdot(n-2)\cdots 2\cdot 1} = \binom{2n}{n} \leq 2^{2n}$$

$$\Rightarrow \quad \pi(2n)-\pi(n) \leq \frac{2n}{\log_2 n}$$

$$\sum_{\substack{\text{PRIMTAL} \\ p\leq n}} \frac{1}{p} \leq 1 + \sum_{i=1}^{\lceil \log_2 n \rceil - 1} \sum_{\substack{\text{PRIMTAL } p \\ 2^i < p \leq 2^{i+1}}} \frac{1}{p} \leq$$

$\leq$

$\leq$

$\square$

2 | 3 | 5 | 7 | 11 | 13 | 17 | 19 | 23 | 29 | 31 | 37 | 41 | 43 | 47 | 53 | 59 | 61 | 67 | 71 | 73 | 79

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DEFINITION

$\pi(n) =$ "ANTAL PRIMTAL $\leq n$"

$\pi(234) = 51$

ALLE PRIMTAL p DIVIDERER $\binom{2n}{n}$ $\quad n < p \leq 2n$

$$n^{\pi(2n)-\pi(n)} \leq \prod_{\substack{\text{PRIMTAL } p \\ n<p\leq 2n}} p \leq \frac{2n\cdot(2n-1)\cdot(2n-2)\cdots(n+2)\cdot(n+1)}{n\cdot(n-1)\cdot(n-2)\cdots 2\cdot 1} = \binom{2n}{n} \leq 2^{2n}$$

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$$\sum_{\substack{\text{PRIMTAL} \\ p\leq n}} \frac{1}{p} \leq 1 + \sum_{i=1}^{\lceil \log_2 n \rceil - 1} \sum_{\substack{\text{PRIMTAL } p \\ 2^i < p \leq 2^{i+1}}} \frac{1}{p} \leq 1 + \sum_{i=1}^{\lceil \log_2 n \rceil - 1} \frac{1}{2^i} \cdot \frac{2^{i+1}}{\log_2 2^i} \leq$$

$\leq$ $\qquad \leq$ $\qquad \square$

2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79
2^1		$4=2^2$		$8=2^3$		$16=2^4$					$32=2^5$			n				$64=2^6$			

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ALLE PRIMTAL $n < p \leq 2n$ DIVIDERER $\binom{2n}{n}$

$$n^{\pi(2n)-\pi(n)} \leq \prod_{\text{PRIMTAL } p,\ n<p\leq 2n} p \leq \frac{2n\cdot(2n-1)\cdot(2n-2)\cdots(n+2)\cdot(n+1)}{n\cdot(n-1)\cdot(n-2)\cdots 2\cdot 1} = \binom{2n}{n} \leq 2^{2n}$$

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$$\sum_{\text{PRIMTAL } p\leq n} \frac{1}{p} \leq 1 + \sum_{i=1}^{\lceil \log_2 n\rceil - 1} \sum_{\text{PRIMTAL } p,\ 2^i<p\leq 2^{i+1}} \frac{1}{p} \leq 1 + \sum_{i=1}^{\lceil \log_2 n\rceil - 1} \frac{1}{2^i}\cdot\frac{2^{i+1}}{\log_2 2^i} \leq 1 + 2\sum_{i=1}^{\lceil \log_2 n\rceil - 1} \frac{1}{i}$$

$\leq \qquad \leq \qquad \square$

2 | 3 | 5 | 7 | 11 | 13 | 17 | 19 | 23 | 29 | 31 | 37 | 41 | 43 | 47 | 53 | 59 | 61 | 67 | 71 | 73 | 79

2^1 $4=2^2$ $8=2^3$ $16=2^4$ $32=2^5$ n $64=2^6$

ALLE PRIMTAL $n < p \le 2n$ DIVIDERER $\binom{2n}{n}$

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$\pi(n) =$ "ANTAL PRIMTAL $\le n$"

$\pi(234) = 51$

$$n^{\pi(2n)-\pi(n)} \le \prod_{\text{PRIMTAL } p,\ n<p\le 2n} p \le \frac{2n\cdot(2n-1)\cdot(2n-2)\cdots(n+2)\cdot(n+1)}{n\cdot(n-1)\cdot(n-2)\cdots 2\cdot 1} = \binom{2n}{n} \le 2^{2n}$$

$$\Rightarrow \quad \pi(2n)-\pi(n) \le \frac{2n}{\log_2 n}$$

$$\sum_{\text{PRIMTAL } p\le n} \frac{1}{p} \le 1 + \sum_{i=1}^{\lceil \log_2 n\rceil - 1} \sum_{\text{PRIMTAL } p,\ 2^i<p\le 2^{i+1}} \frac{1}{p} \le 1 + \sum_{i=1}^{\lceil \log_2 n\rceil - 1} \frac{1}{2^i}\cdot\frac{2^{i+1}}{\log_2 2^i} \le 1 + 2\sum_{i=1}^{\lceil \log_2 n\rceil - 1} \frac{1}{i}$$

$$\le 1 + 2\left(\lfloor \log_2\left(\lceil \log_2 n\rceil - 1\right)\rfloor + 1\right) \le \square$$

i	1	2	3	4	5	6
2^i	2	4	8	16	32	64

2 | 3 | 5 | 7 | 11 | 13 | 17 | 19 | 23 | 29 | 31 | 37 | 41 | 43 | 47 | 53 | 59 | 61 | 67 | 71 | 73 | 79

$4 = 2^2$, $8 = 2^3$, $16 = 2^4$, $32 = 2^5$, n, $64 = 2^6$

ALLE PRIMTAL $n < p \le 2n$ DIVIDERER $\binom{2n}{n}$

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$\pi(234) = 51$

$$n^{\pi(2n)-\pi(n)} \le \prod_{\text{PRIMTAL } p,\ n<p\le 2n} p \le \frac{2n\cdot(2n-1)\cdot(2n-2)\cdots(n+2)\cdot(n+1)}{n\cdot(n-1)\cdot(n-2)\cdots 2\cdot 1} = \binom{2n}{n} \le 2^{2n}$$

$$\Rightarrow \quad \pi(2n)-\pi(n) \le \frac{2n}{\log_2 2n}$$

$$\sum_{\text{PRIMTAL } p\le n} \frac{1}{p} \le 1 + \sum_{i=1}^{\lceil \log_2 n\rceil - 1} \sum_{\text{PRIMTAL } p,\ 2^i<p\le 2^{i+1}} \frac{1}{p} \le 1 + \sum_{i=1}^{\lceil \log_2 n\rceil - 1} \frac{1}{2^i}\cdot\frac{2^{i+1}}{\log_2 2^i} \le 1 + 2\sum_{i=1}^{\lceil \log_2 n\rceil - 1} \frac{1}{i}$$

$$\le 1 + 2\left(\lfloor \log_2(\lceil \log_2 n\rceil - 1)\rfloor + 1\right) \le 3 + 2\log_2\log_2 n \qquad \square$$

2	3	5	7	11	13	17	19	23	29	31	37	41	43	47	53	59	61	67	71	73	79
2^1	$4=2^2$		$8=2^3$		$16=2^4$						$32=2^5$			n				$64=2^6$			