Purely Functional Worst Case Constant Time Catenable Sorted Lists

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Joint work with

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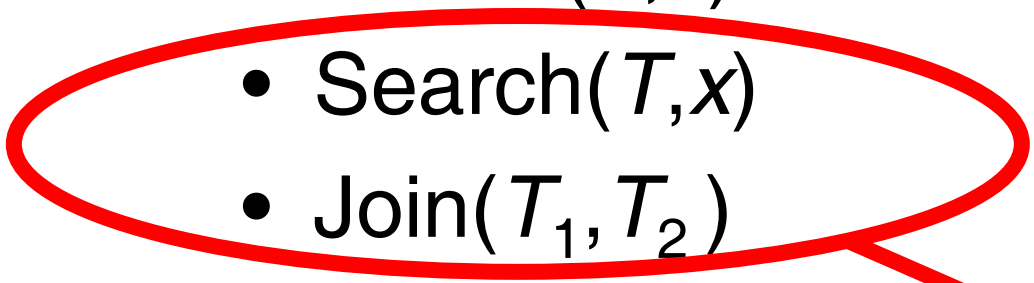
Catenable Sorted Lists

- Insert(T,x)
- Delete(T,x)
- Search(T,x)
- Join(T_1, T_2)
- Split(T,x)



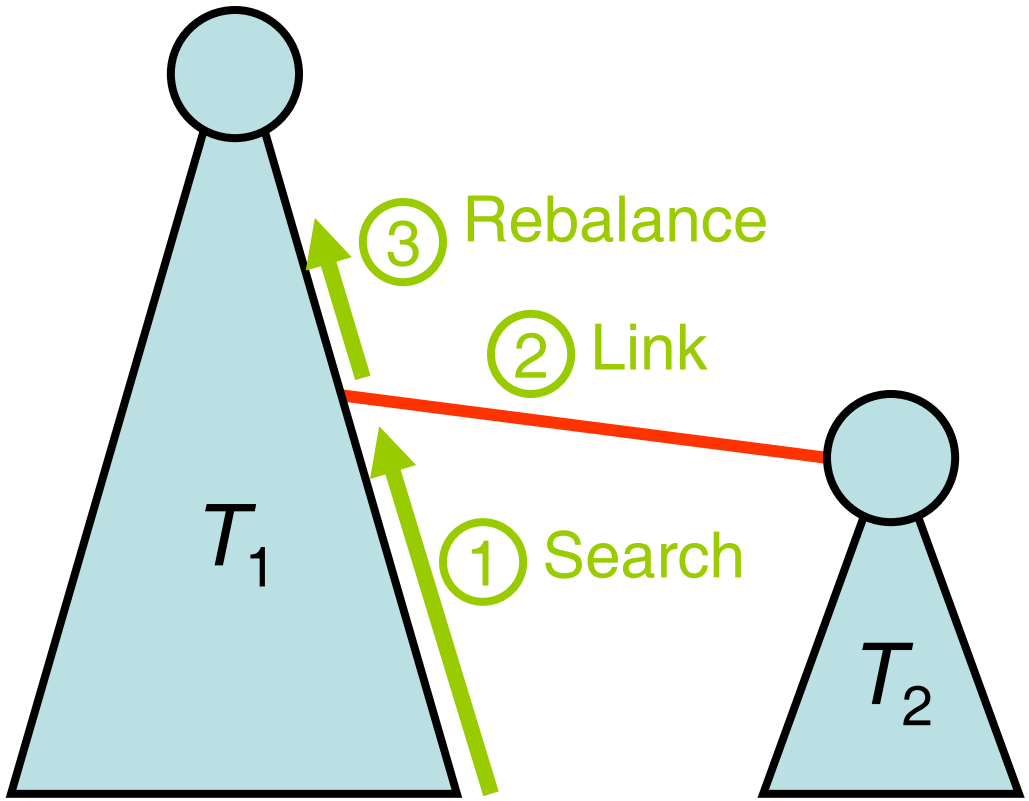
This talk

T_1 = 2,5,7,8,10,11 T_2 = 13,17,19,34,58,79

Join(T_1, T_2) = 2,5,7,8,10,11,13,17,19,34,58,79

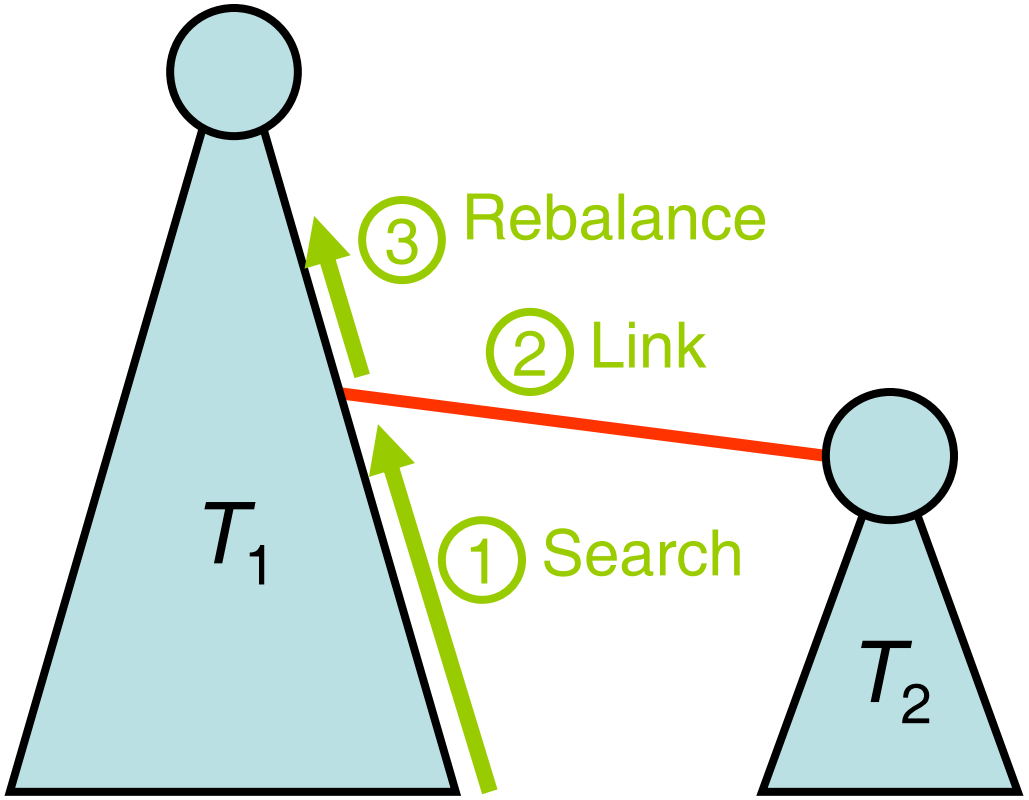
Catenable Sorted Lists: Search Trees

(2,4-trees, red-black trees…)



Catenable Sorted Lists: Search Trees

(2,4-trees, red-black trees…)



1. Worst-case O(log $|T_2|$)
   or O(loglog $|T_2|$)
   or amortized O(1)
2. Worst-case O(1)
3. Worst-case O(log $|T_1|$)
   or amortized O(1)
   or worst-case O(1)

Catenable Sorted Lists

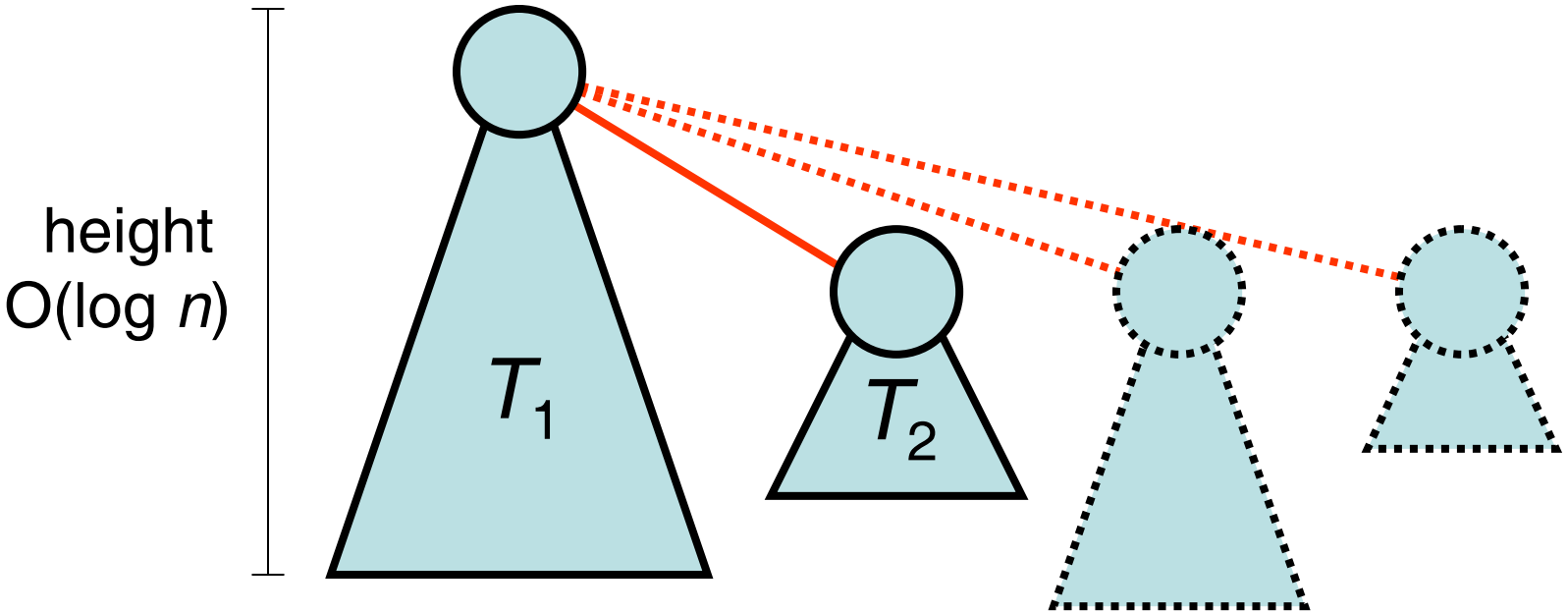
	Search trees	Kaplan Tarjan'96	This talk
Search	log n	log n	log n
Insert/Delete	log n	log n	log n
Join	log n 1*	loglog n_s	1
Split	log n	log n_s	-

$n = |T|$ $n_s = \min(|T_1|,|T_2|)$ * amortized

O(1) Join and O(log *n*) Search

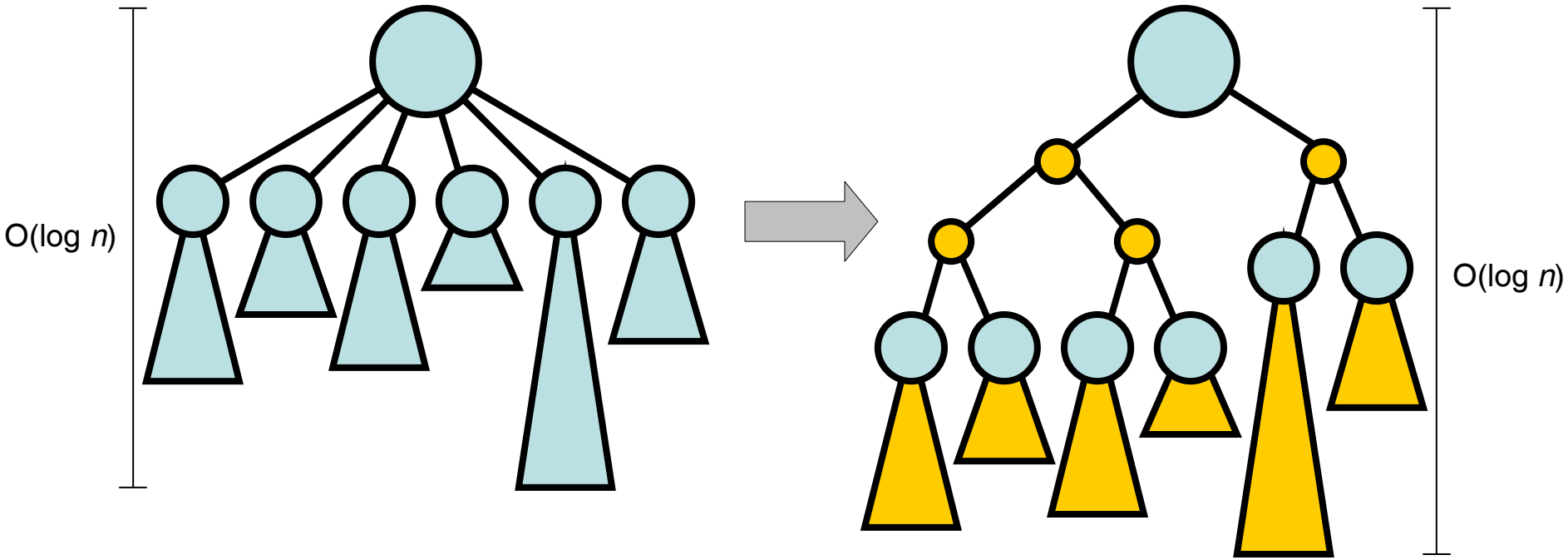
The ideas….

Idea 1: Linking By Size



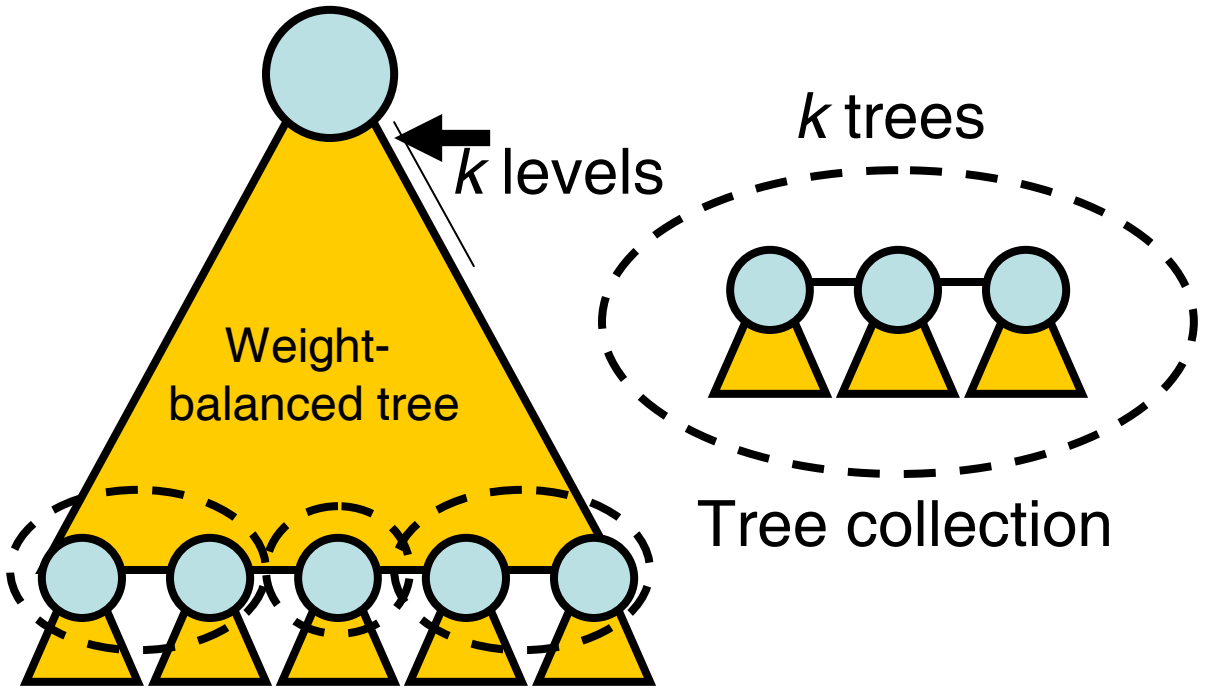
Problem: Nodes of arbitrary degree

Idea 2: Represent Nodes by Weight-Balanced Trees

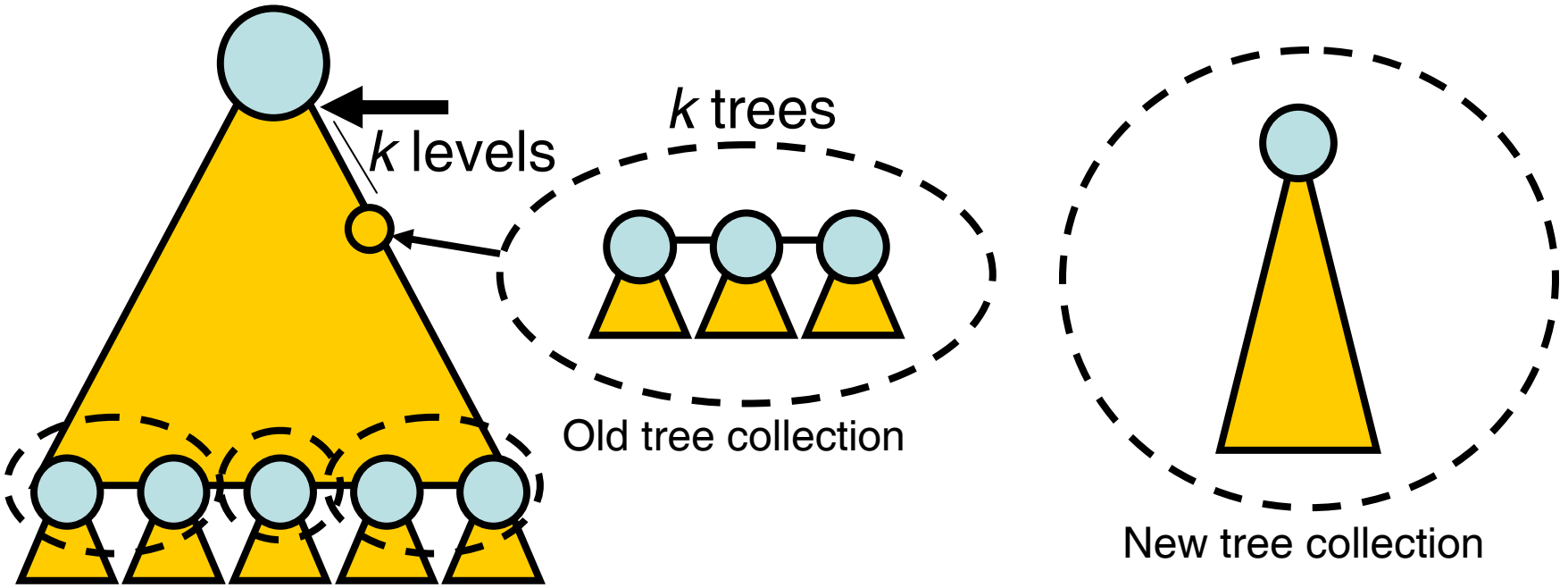


Problem: Does not support linking by size in O(1) time

Idea 3: Tree-Collections and Lazy Join



Idea 3 (*cont.*): Lazy Join

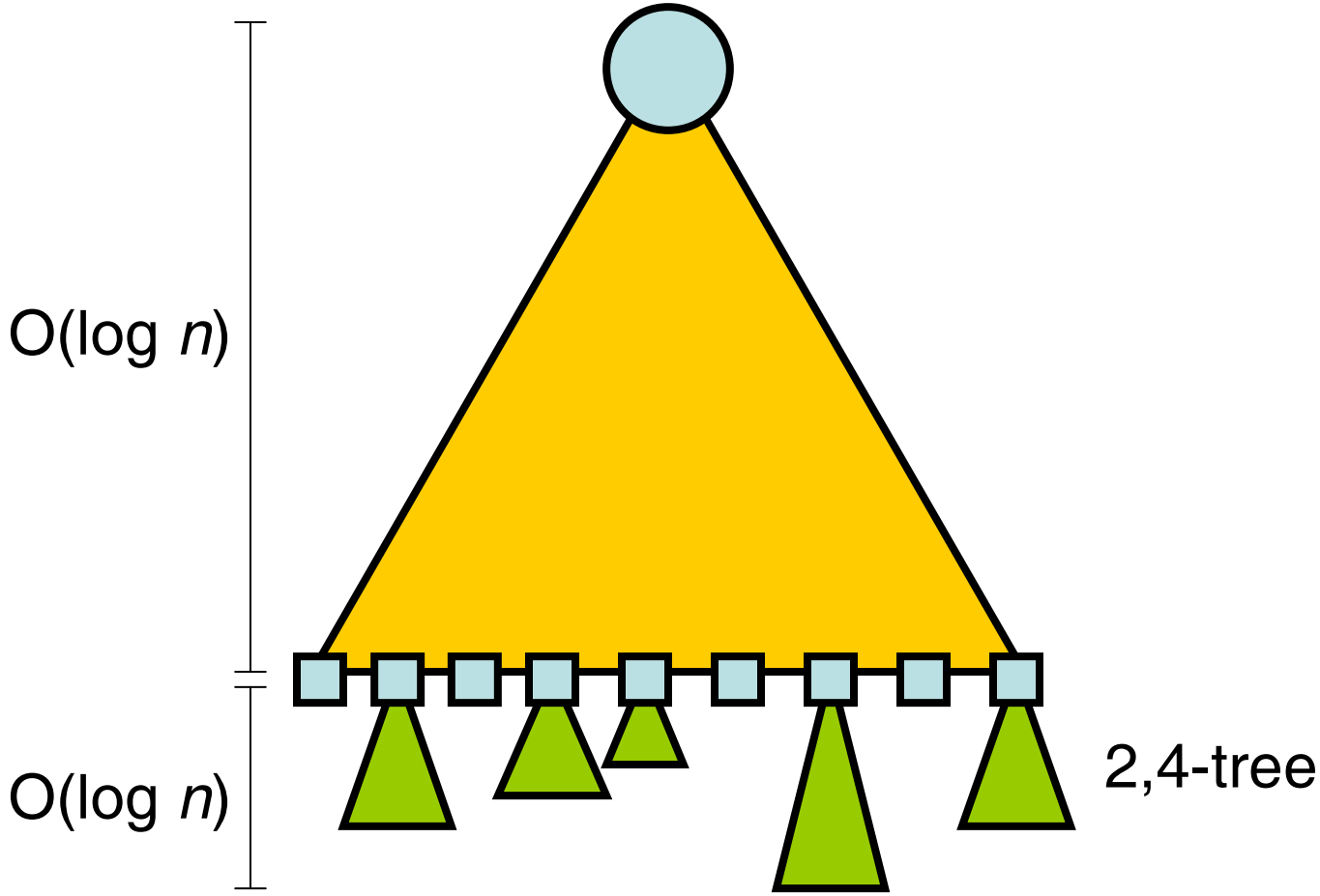


Weight balanced trees with top-down rebalancing
[Bent, Sleator, Tarjan 1985]

Catenable Sorted Lists

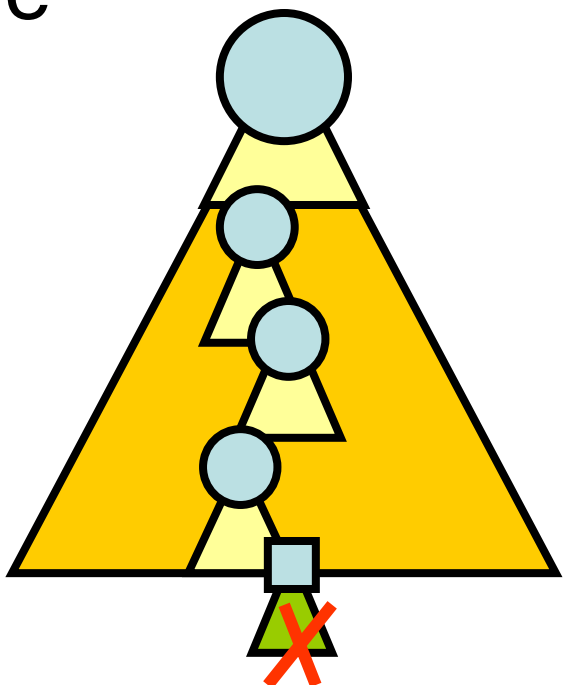
Operation	Worst-case time
Search	O(log *n*)
Join	O(1)

Supporting Insertions: Buckets



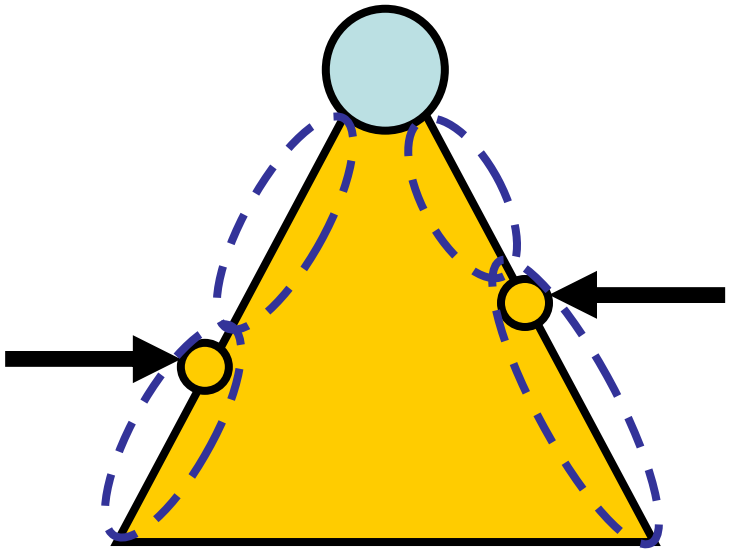
Supporting Deletions: The ideas…

- Delete leaf
- Rebalance weight-balanced trees on root-to-leaf path
- Join O(1) weight-balanced trees in O(log *n*) time



Functional Catenable Sorte Lists

- Functional setting = no side effects allowed
- Data structures are automatically persistent
- Idea: Represent spines by functional catenable lists [Kaplan, Tarjan 1995]



Conclusion

	Search trees	This talk ♪
Search	log *n*	log *n*
Insert/Delete	log *n*	log *n*
Join	log *n* 1*	1
Split	log *n*	-

* amortized ♪ purely functional