Optimal Finger Search Trees in the Pointer Machine

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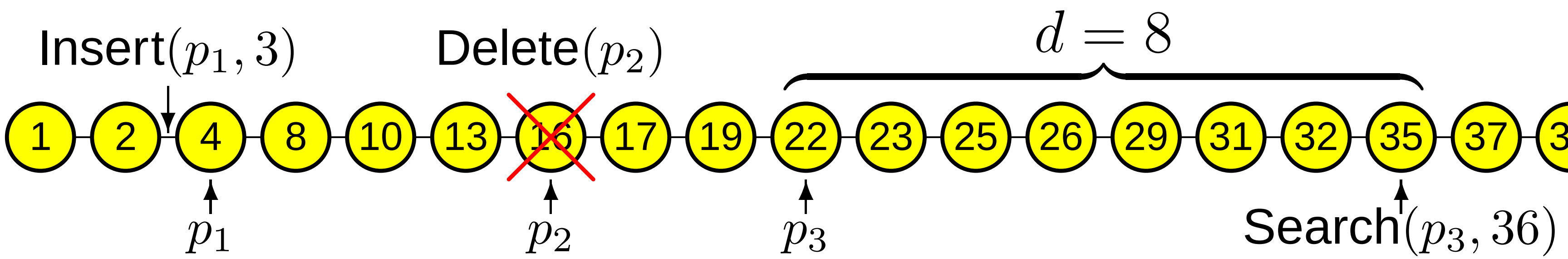
Finger Search Trees



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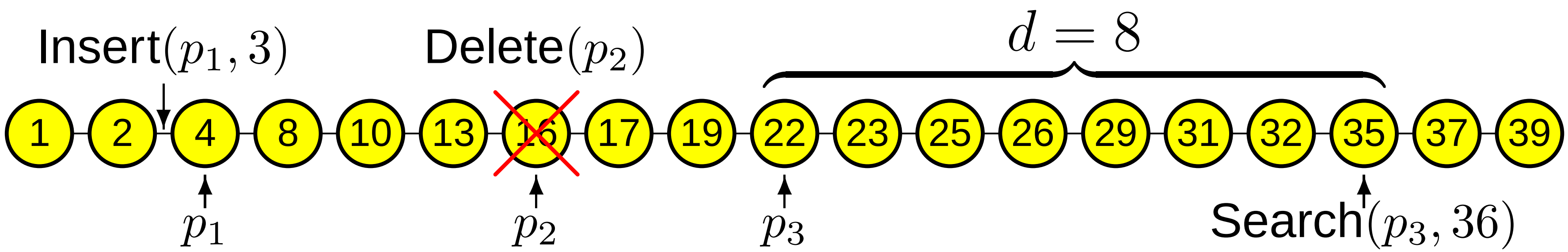
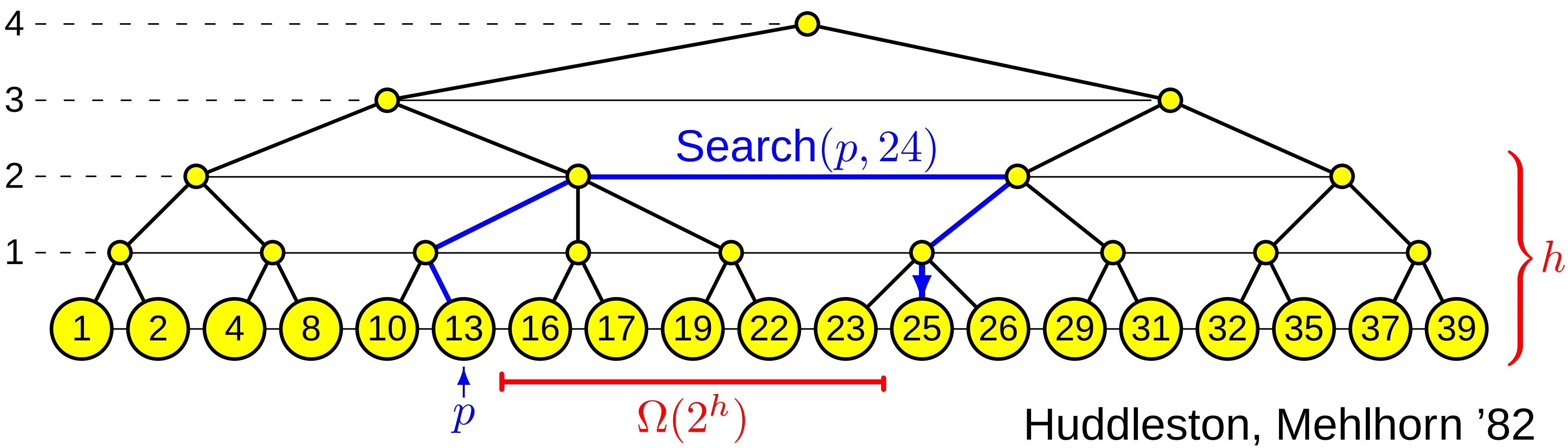
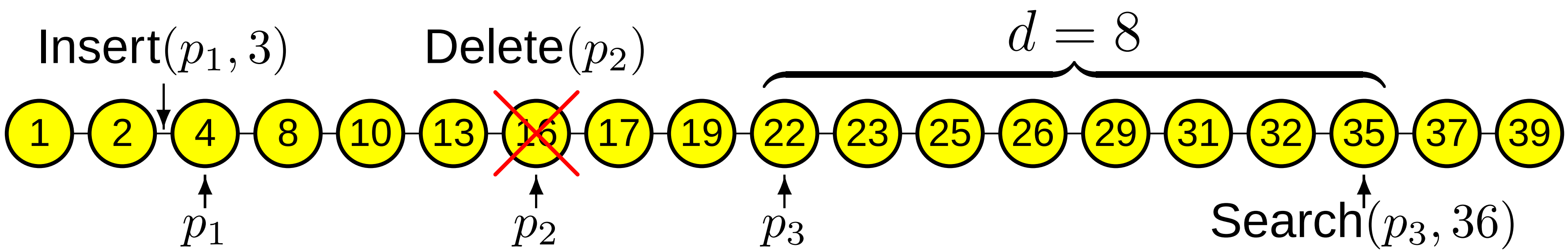


Insert, Delete	$O(1)$
Search	$O(\log d)$



Huddleston, Mehlhorn '82

Finger Search Trees



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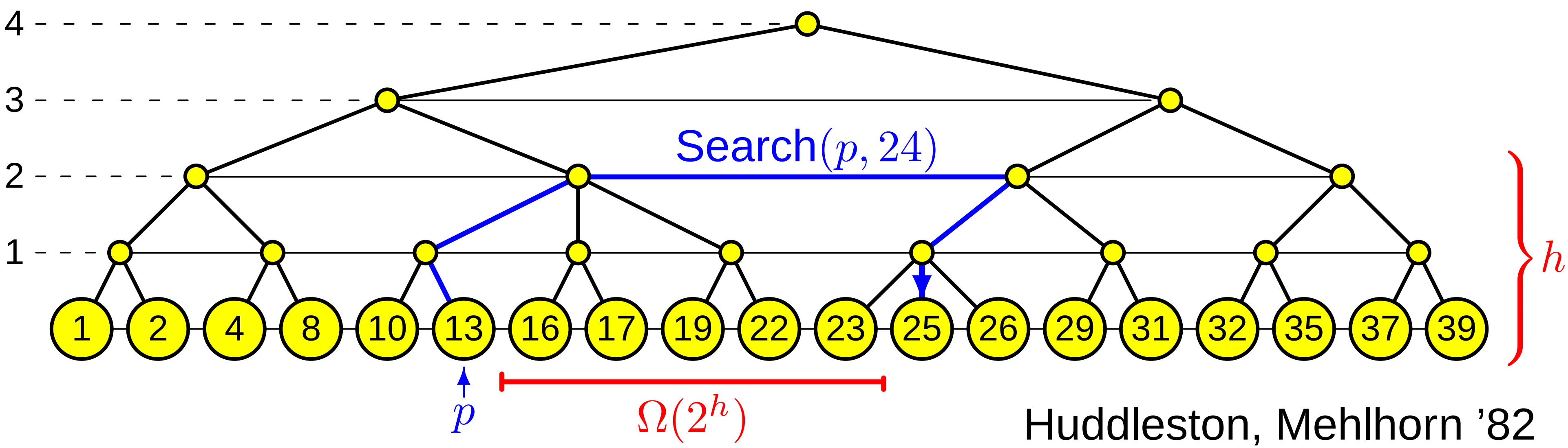
Amortized



Huddleston, Mehlhorn ’82

History

<table>
<tr><th></th><th>Insert</th><th>Delete</th><th>Search</th></tr>
<tr><td>AVL-trees, (2,3)-trees [a)]</td><td colspan="2">$\log n$</td><td>$\log n$</td></tr>
<tr><td>Red-black-trees, (2,4)-trees [a)]</td><td colspan="2">1[b)]</td><td>$\log n$</td></tr>
<tr><td>Levcopoulos, Overmars ’88 [a)]</td><td colspan="2">1</td><td>$\log n$</td></tr>
<tr><td>Guibas et al. ’77, Tsakalidis ’85 [a)]</td><td colspan="2">1[c)]</td><td>$\log d$</td></tr>
<tr><td>Harel, Lucker ’79 [a)]</td><td colspan="2">$\log^* n$</td><td>$\log d$</td></tr>
<tr><td>Huddleston, Mehlhorn ’82 [a)]</td><td colspan="2">1[b)]</td><td>$\log d$</td></tr>
<tr><td>Brodal ’98 [a)]</td><td>1</td><td>$\log^* n$</td><td>$\log d$</td></tr>
<tr><td>This talk [a)]</td><td colspan="2">1</td><td>$\log d$</td></tr>
<tr><td>Dietz, Raman ’94 [d)]</td><td colspan="2">1</td><td>$\log d$</td></tr>
<tr><td>Andersson, Thorup ’00 [e)]</td><td colspan="2">1</td><td>$\sqrt{\frac{\log d}{\log \log d}}$</td></tr>
</table>

[a)] Pointer machine [b)] Amortized [c)] $O(1)$ movable fingers

[d)] Comparison RAM [e)] Word RAM

This Talk

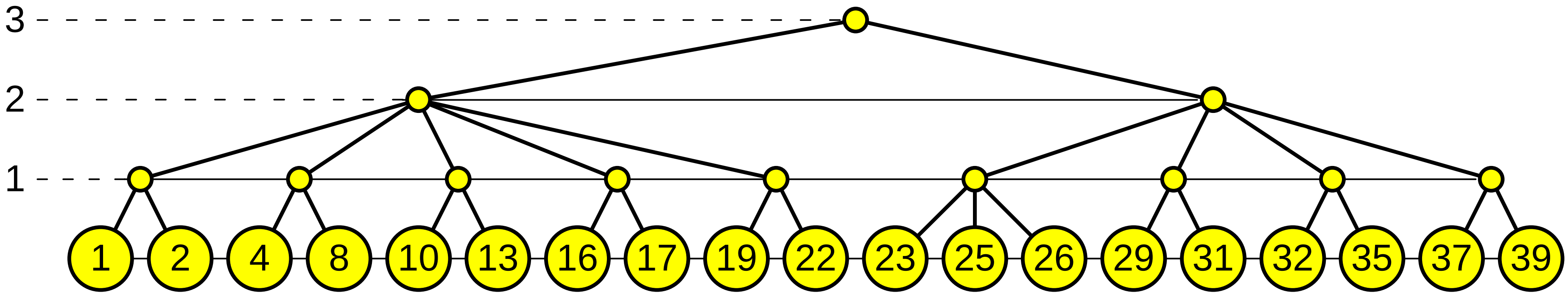
Insert	$O(1)$
Delete	$O(1)$
Search	$O(\log d)$

Worst-case

Pointer machine

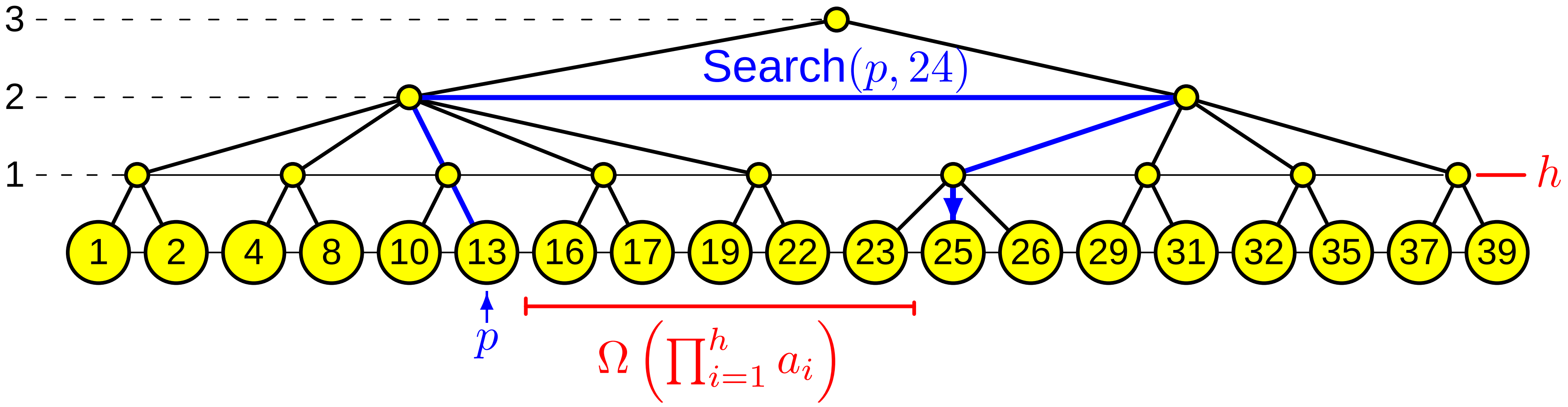
- Level dependent degrees (Andersson '96)
- Components (Brodal '96)
- Multi-split and multi-join
- Incremental (pre)processing

Level Dependent Degrees



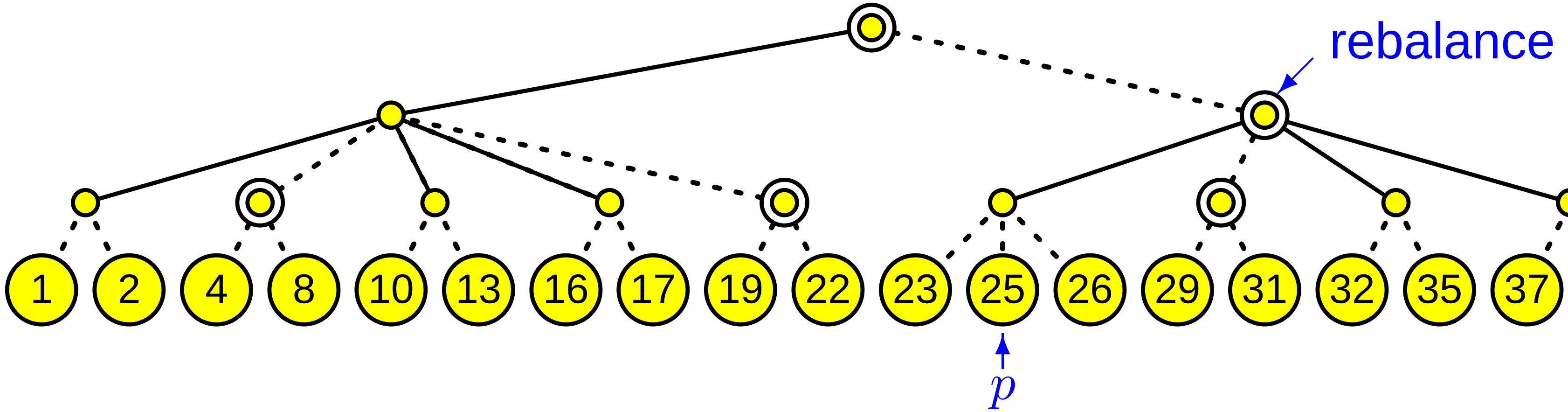
- Level i nodes (except root) have degree $[\, a_i : {a_i}^3 \,]$
- Each node represented by Levcopoulos, Overmars ’88

Level Dependent Degrees



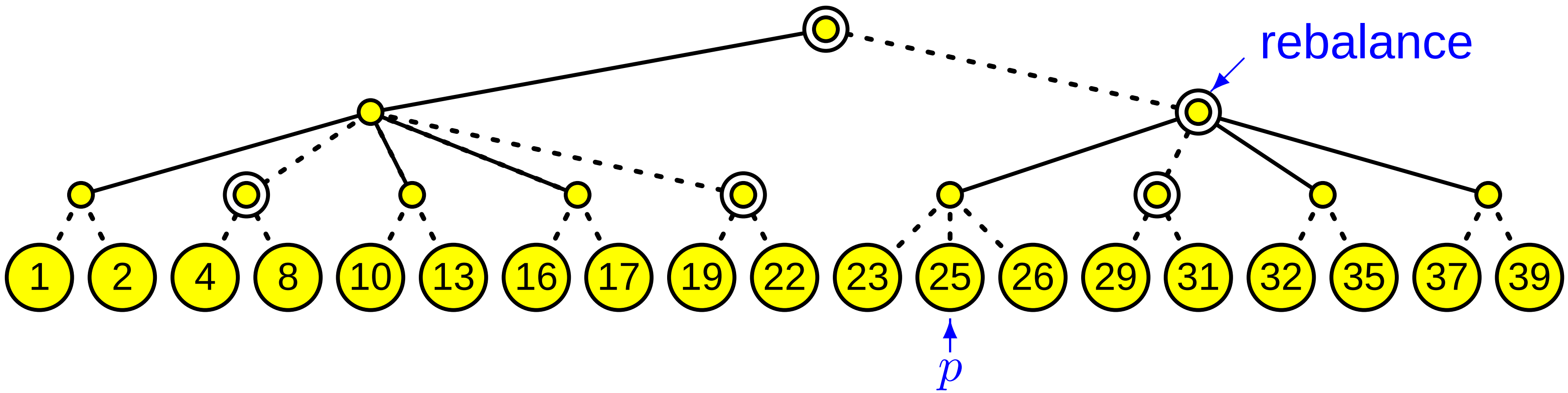
- Level i nodes (except root) have degree $[\, a_i : {a_i}^3 \,]$
- Each node represented by Levcopoulos, Overmars '88
- Search $O(\sum_{i=1}^{h+1} \log a_i) = O(\log d)$ if $a_{h+1} = a_h^{O(1)}$

Components



- Mechanisme to identify where to rebalance
- Components = partition of internal nodes into subtrees

Components



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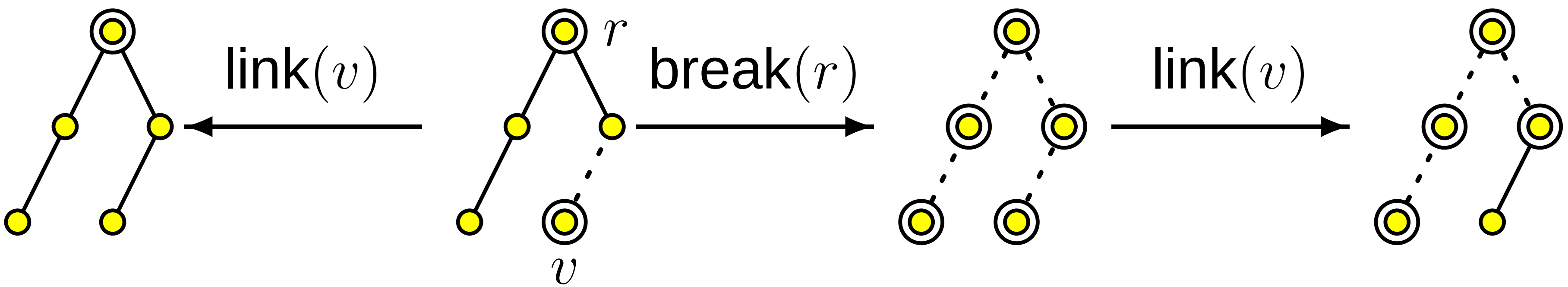
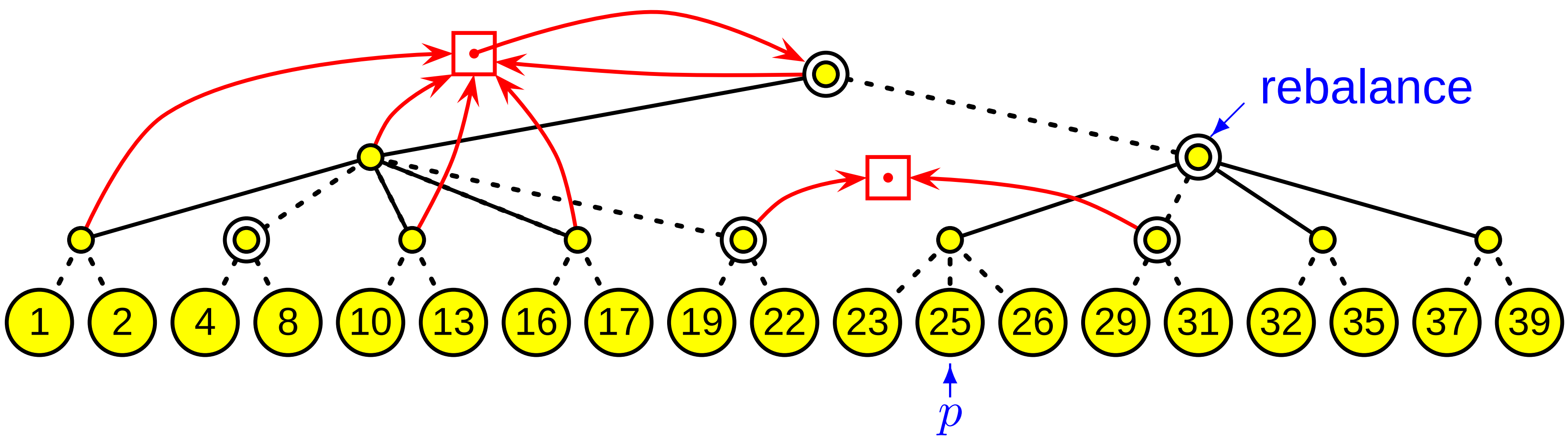
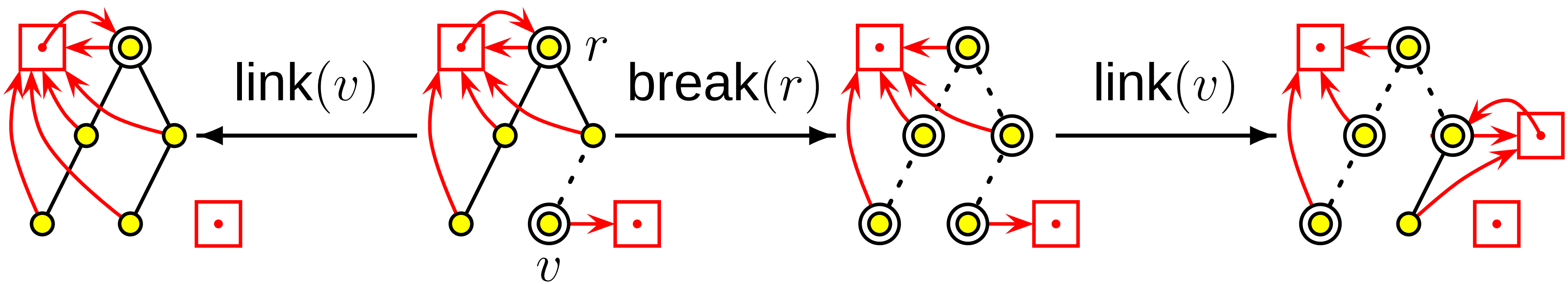


Components



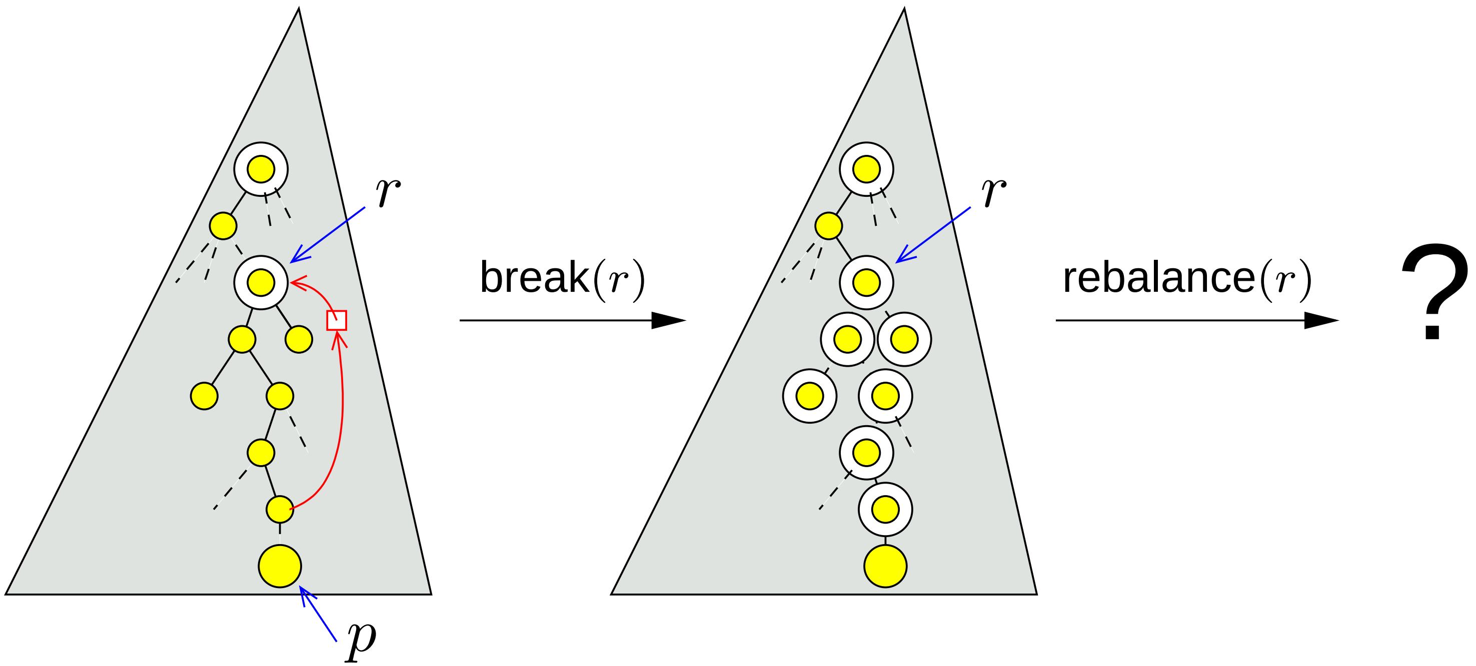
- Mechanisme to identify where to rebalance
- Components = partition of internal nodes into subtrees
- component records with root pointers (nil if singleton)



Updates

Insert(p, e)
r = component root
create-leaf(p, e)
break(r)
rebalance(r)

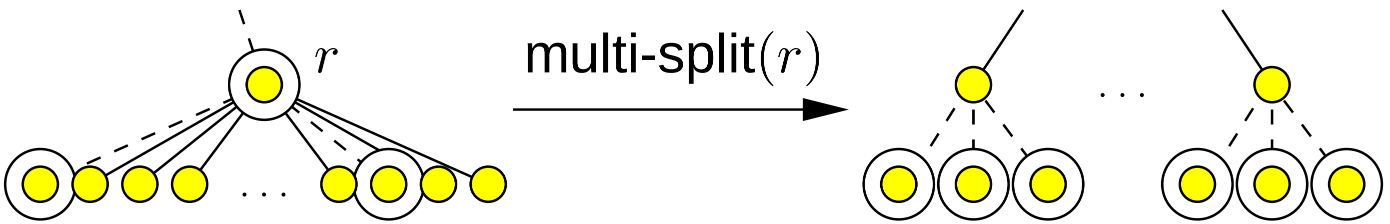
Delete(p)
r = component root
remove-leaf(p)
break(r)
rebalance(r)



rebalance(r)

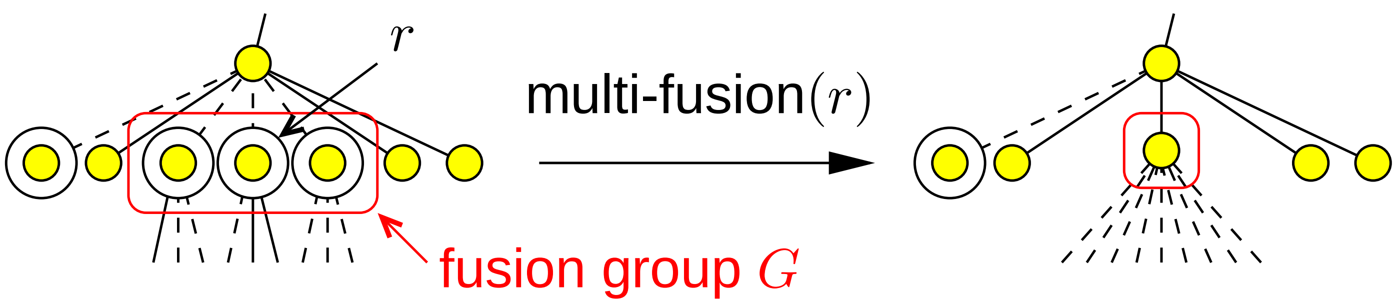
degree$(r) > 3{a_i}^2$

- link(r)
- split r into at most a_i nodes of degree $[{a_i}^2 : 3{a_i}^2]$



degree$(r) < {a_i}^2$

- $\forall v \in G$: break(v), G total degree $[{a_i}^2 : 3{a_i}^2]$
- fusion r with all nodes in G
- link(r)

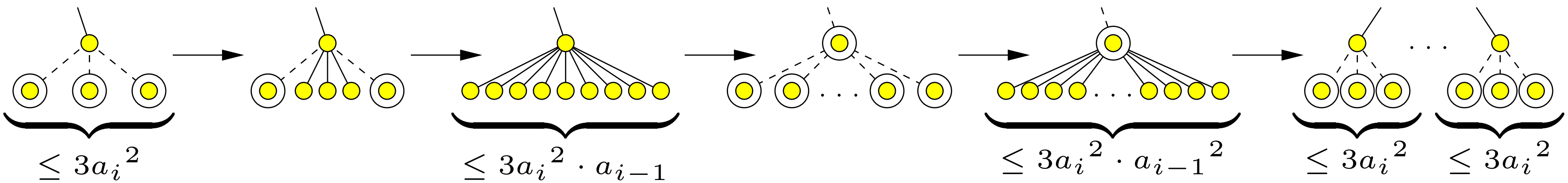


Proof of Degree $[a_i : {a_i}^3]$

Assumption : Nodes at level $i-1$ have degree $[a_{i-1} : {a_{i-1}}^3]$

$\leq {a_i}^3$

- split at level $i-1$ generates $\leq a_{i-1}$ nodes
- degree at level $i \leq 3{a_i}^2 \cdot {a_{i-1}}^2 \leq {a_i}^3$

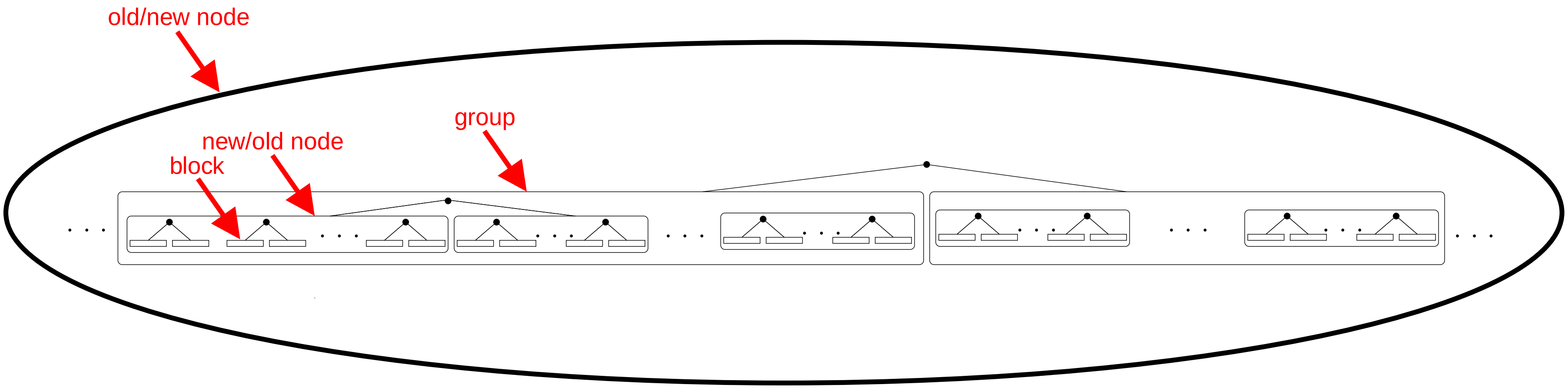


$\geq a_i$

- fusion at level $i-1$ fusions $\leq 3a_{i-1}$ nodes
- degree at level $i \geq {a_i}^2/(3a_{i-1})^2 = a_i$

$\boxed{a_i = 9 \cdot {a_{i-1}}^2}$

Incremental Processing



Internal representation of a node

- Blocks (pairs of)
- Nodes (pairs of)
- Splitting groups
- Fusion groups (pairs of)
- Fusion/splitting group records

Conclusion and Open Problems

Insert	$O(1)$
Delete	$O(1)$
Search	$O(\log d)$

Worst-case

Pointer machine

- Simpler construction ?
- Direct binary tree solution ?
- (a, b)-trees with worst-case update time ?
- Components usefull for deamortization of other structures ?