Algorithms and Data Structures
for Faulty Memory

Computer Science Day 2008

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MADALGO
CENTER FOR MASSIVE DATA ALGORITHMICS

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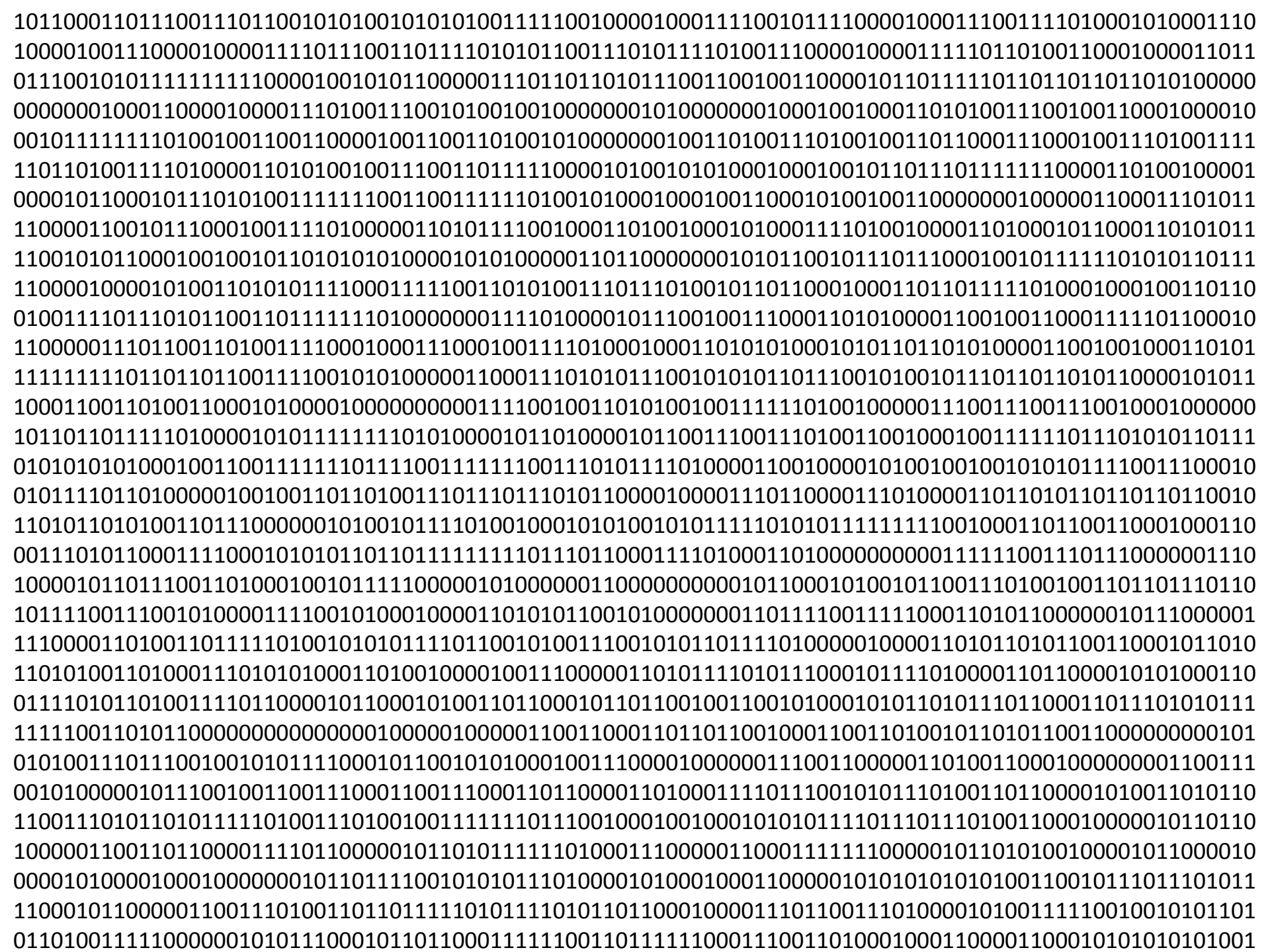
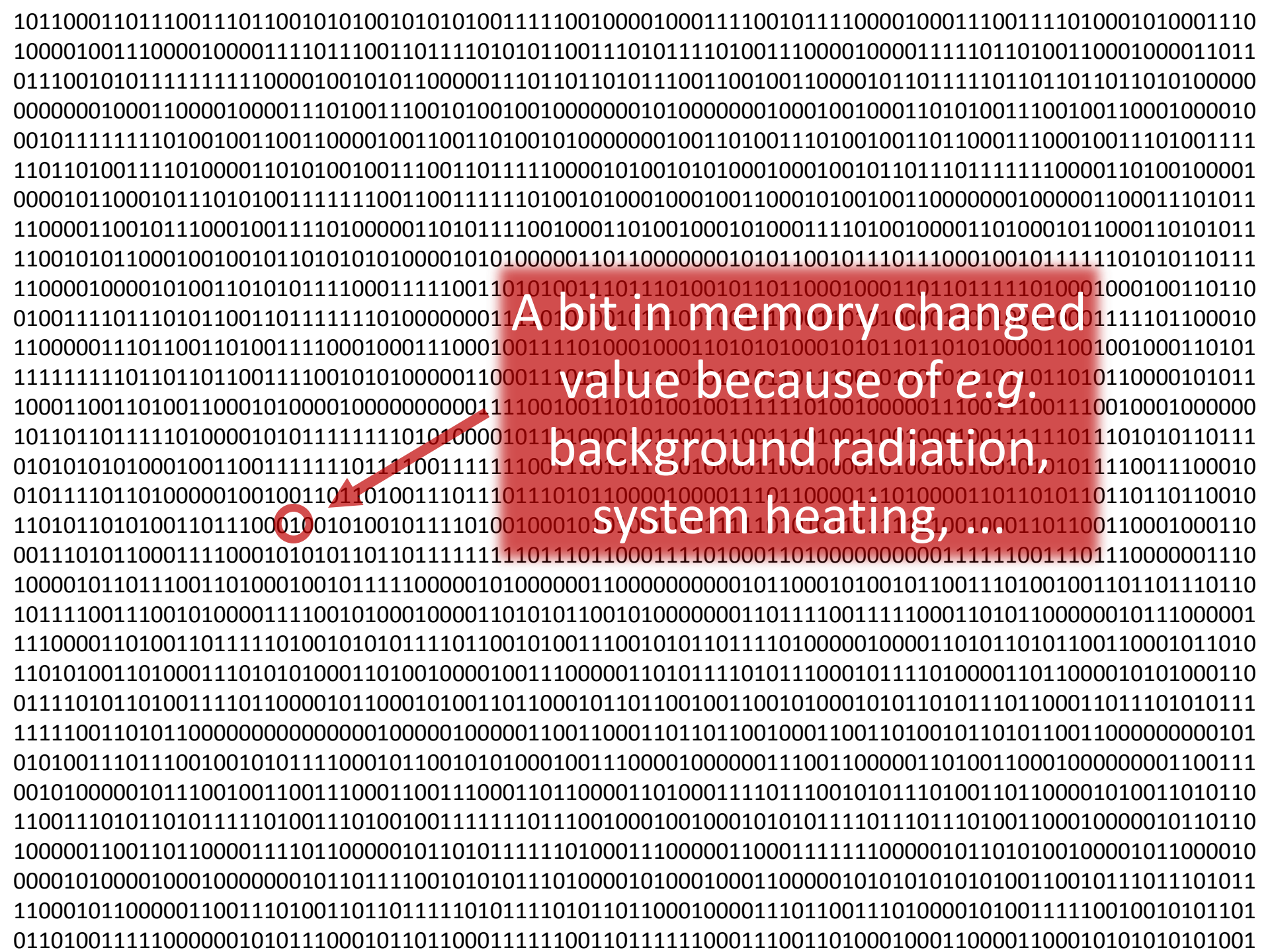
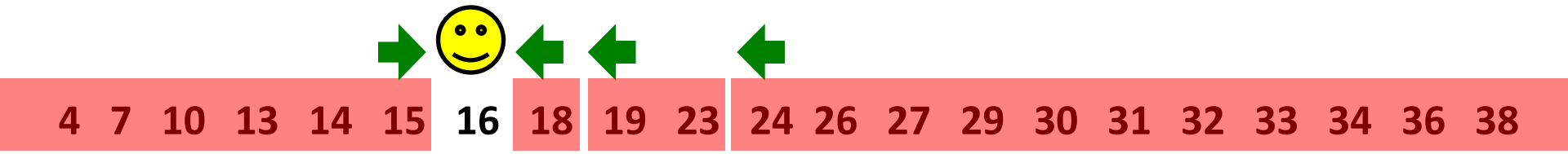
"You have to provide reliability on a software level. If you're running 10,000 machines, something is going to die every day."

— Google fellow Jeff Dean



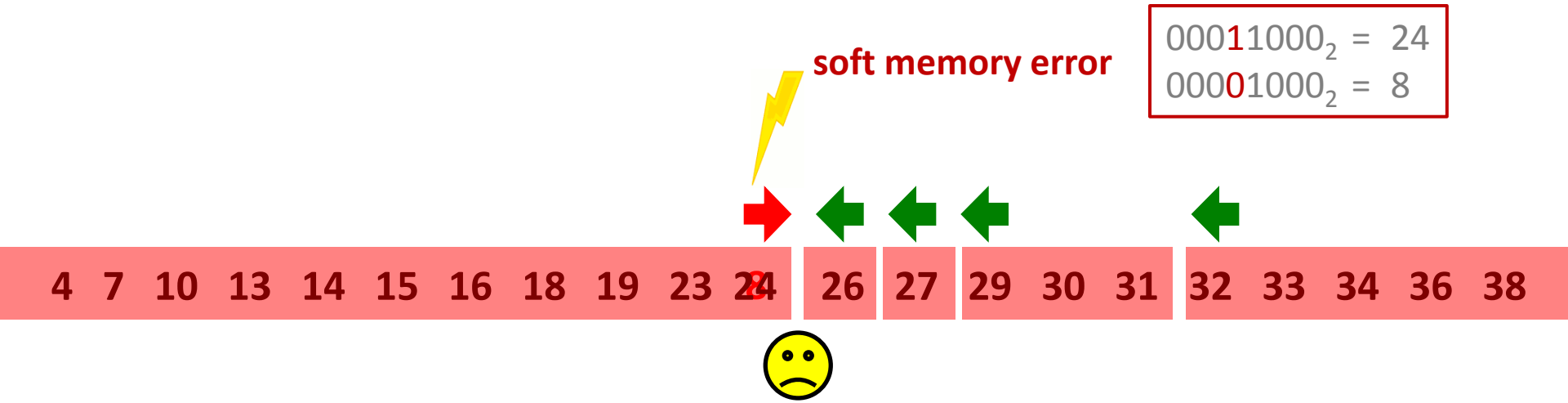

A bit in memory changed value because of *e.g.* background radiation, system heating, ...

Binary Search for 16



O(log *N*) comparisons

Binary Search for 16



Requirement: If the search key ocours in the array as an uncorrupted value, then we should report a match *!*

Where is Kurt ?

Where is Kurt ?

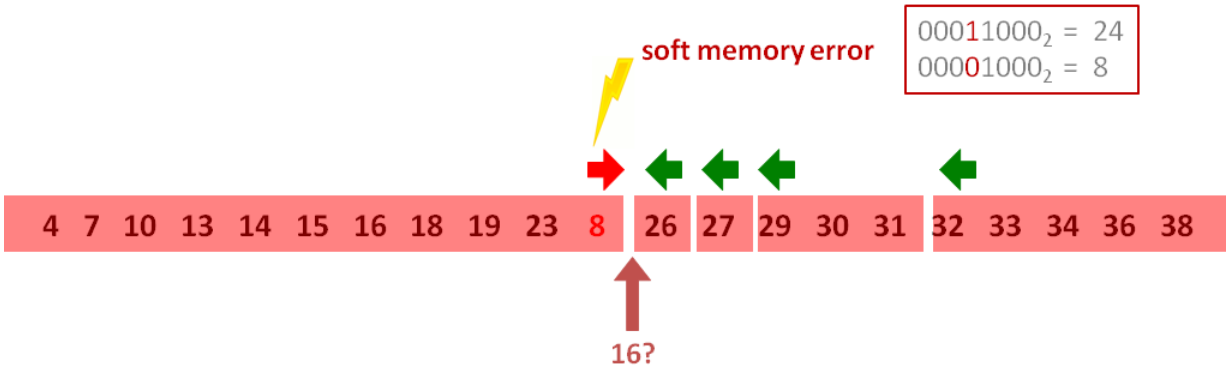
Where is Kurt ?

If at most 4 faulty answers then Kurt is somewhere here

Faulty-Memory RAM Model

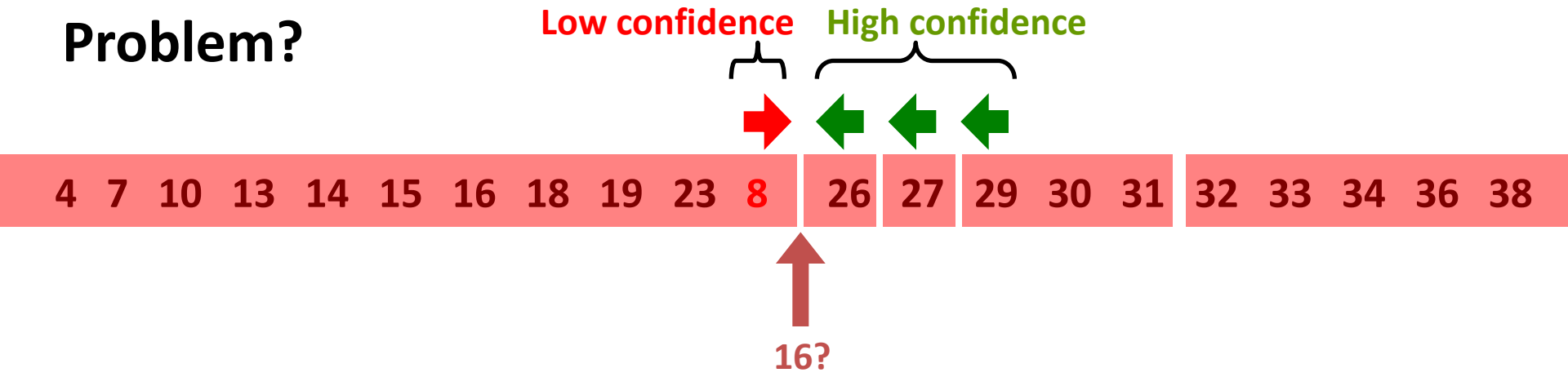
Finocchi and Italiano, STOC'04

- Content of memory cells can get corrupted
- Corrupted and uncorrupted content cannot be distinguished
- *O*(1) safe registers
- **Assumption**: At most δ corruptions



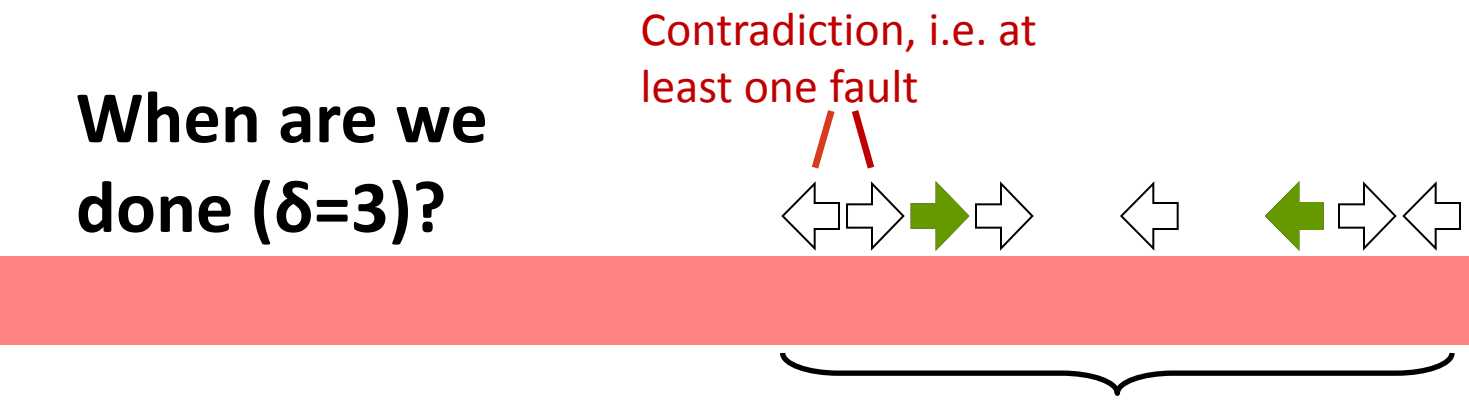
Faulty-Memory RAM: Searching

Problem?



Faulty-Memory RAM: Searching

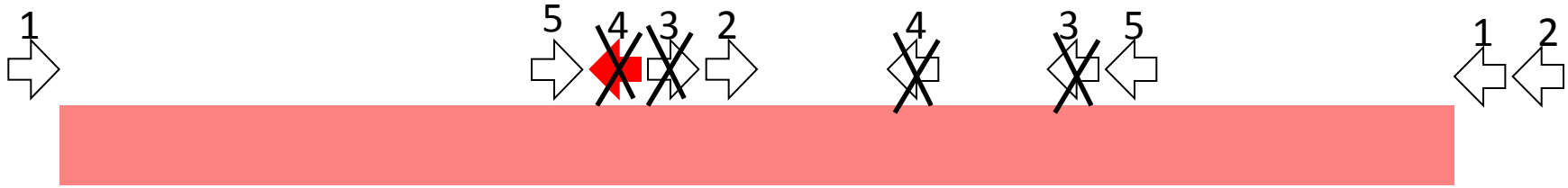
When are we done (δ=3)?



If range contains at least δ+1 ⇨ and δ+1 ⇦ then there is at least one uncorrupted ➡ and ⬅, i.e. *x* must be contained in the range

Faulty-Memory RAM: Θ(log *N* + δ) Searching

Brodal, Fagerberg, Finocchi, Grandoni, Italiano, Jørgensen, Moruz, Mølhave, ESA'07



If verification fails

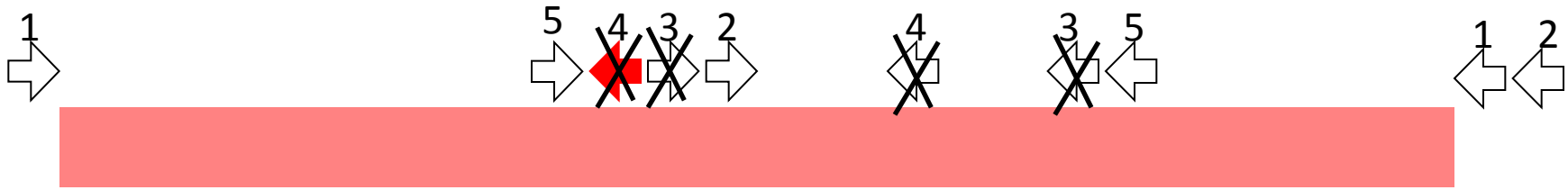
→ contradiction, i.e. ≥1 memory-fault

→ ignore 4 last comparisons

→ backtrack one level of search

Faulty-Memory RAM: Θ(log *N* + δ) Searching

Brodal, Fagerberg, Finocchi, Grandoni, Italiano, Jørgensen, Moruz, Mølhave, ESA'07

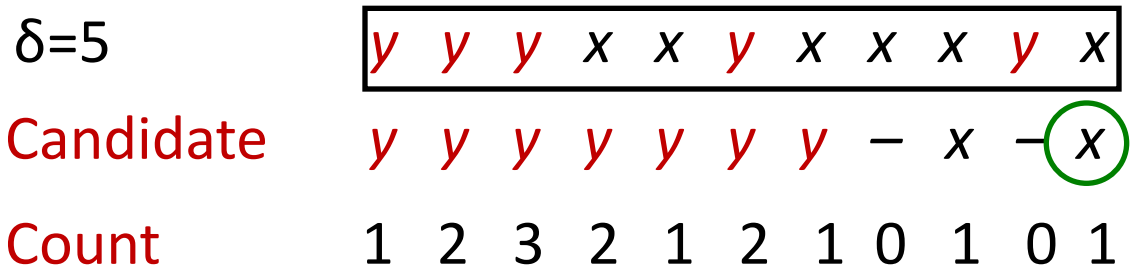


- Standard binary search + verification steps
- At most δ verification steps can fail/backtrack
- **Detail**: Avoid repeated comparison with the same (wrong) element by grouping elements into blocks of size $O(\delta)$

Faulty-Memory RAM: Reliable Values

- Store 2δ+1 copies of value x - at most δ copies uncorrupted
- x = majority
- Time $O(\delta)$ using two safe registers (candidate and count)

Boyer and Moore '91

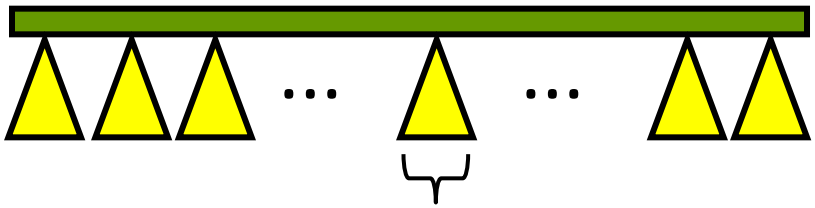


Faulty-Memory RAM: Dynamic Dictionaries

Brodal, Fagerberg, Fnocchi, Grandoni, Italiano, Jørgensen, Moruz, Mølhave, ESA'07

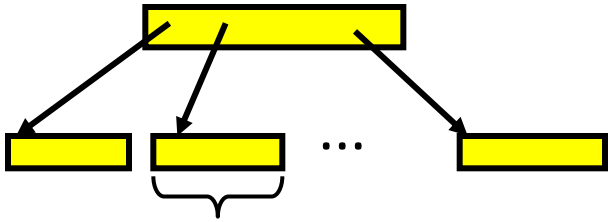
- Packed array
- Reliable pointers and keys
- Updates $O(\delta \cdot \log^2 N)$
- Searches = fault tolerant $O(\log N+\delta)$

Itai, Konheim, Rodeh, 1981



$\Theta(\delta \cdot \log N)$ elements

- 2-level buckets of size $O(\delta \cdot \log N)$
- Root: Reliable pointers and keys
- Bucket search/update amortized $O(\log N+\delta)$



$\Theta(\delta)$ elements

- **Search and update amortized $O(\log N+\delta)$**

Fault-Tolerant Results

- Merging, time $\Theta(N+\delta^2)$ — Finocchi, Grandoni, Italiano, ICALP'06
- Priority queue, time $\Theta(\log N+\delta)$ — Jørgensen, Moruz, Mølhave, WADS'07
- Sorting, time $\Theta(N \cdot \log N+\delta^2)$ — Finocchi, Grandoni, Italiano, ICALP'06
- Static and dynamic dictionary, time $\Theta(\log N+\delta)$
  Brodal, Fagerberg, Finocchi, Grandoni, Italiano, Jørgensen, Moruz, Mølhave, ESA'07
  Finocchi, Grandoni, Italiano, ICALP'06
- External-memory fault tolerant searching, $\Theta\left(\frac{1}{\varepsilon}\log_B N+\frac{\delta}{B^{1-\varepsilon}}\right)$ I/Os
  Brodal, Jørgensen, Mølhave, Submitted

Allan G. Jørgensen and Gerth S. Brodal at MFCS'07