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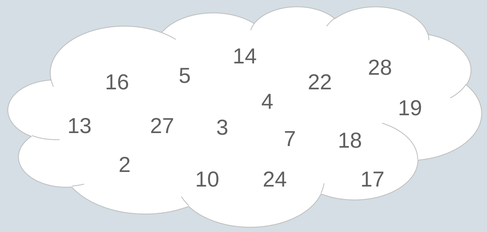
Cache-Oblivious Dynamic Dictionaries with Update/Query Tradeoffs

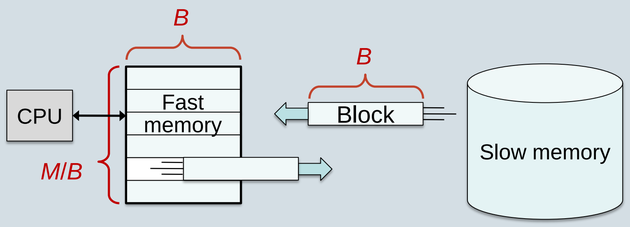
Problem and Results

Dictionary



- Maintain a set of elements
- Search(x) – return the largest element $\leq x$ in the set (static and dynamic version)
- Insert new / delete existing element (dynamic version)

Models



I/O Model
[Aggarwal, Vitter 1988]

- Fast internal memory, size M
- Slow memory, blocks of size B
- Cost = number of block transfers (I/Os) between the two memories

Cache-oblivious Model
[Frigo, Leiserson, Prokop, Ramachandran 1999]

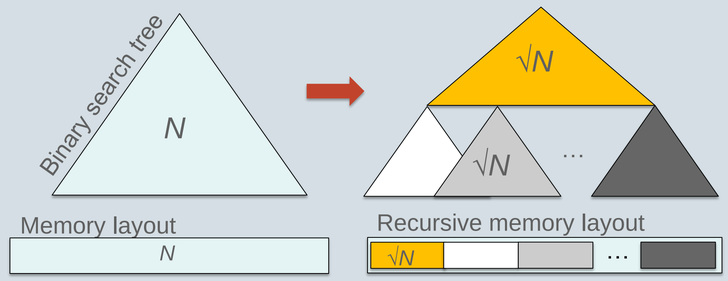
- Algorithms are not parameterized by B and M
- Analyzed in the **ideal-cache model,** same as I/O model but using optimal cache replacement strategy
- Algorithms automatically apply to multi-level hierarchies

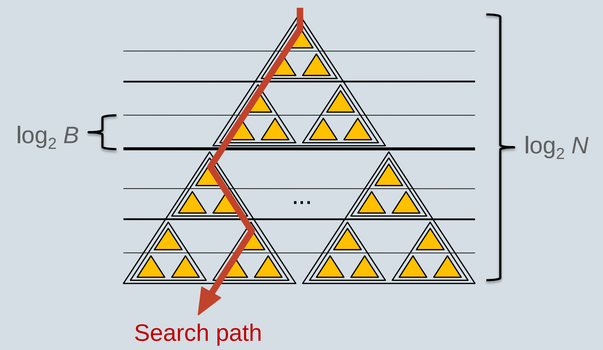
	Search	Updates	
Bayer, McCreight 1972	$O(\log_B N)$	$O(\log_B N)$	I/O Model
Brodal, Fagerberg 2003	$O(\frac{1}{\varepsilon} \cdot \log_B N)$	$O(\frac{1}{\varepsilon B^{\varepsilon}} \cdot \log_B N)$	I/O Model
Prokop 1999	$O(\log_B N)$	N/A	Cache-oblivious Model
Brodal, Jacob, Fagerberg 2002	$O(\log_B N)$	$O(\log_B N)$	Cache-oblivious Model
Brodal, Demaine, Iacono, Fineman, Langerman, Munro 2010	$O(\frac{1}{\varepsilon} \cdot \log_B N)$	$O(\frac{1}{\varepsilon B^{\varepsilon}} \cdot \log_B N)$	Cache-oblivious Model

Construction Ideas

Static Dictionaries [Prokop 1999]

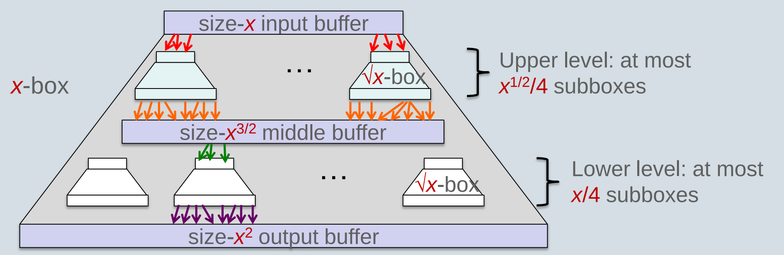
- Perform a recursive memory layout of a perfect binary search tree
- Subtrees of height $O(\log_2 B)$ fit into $O(1)$ blocks, i.e. search cost becomes $O(\log_2 N / \log_2 B) = O(\log_B N)$ I/Os

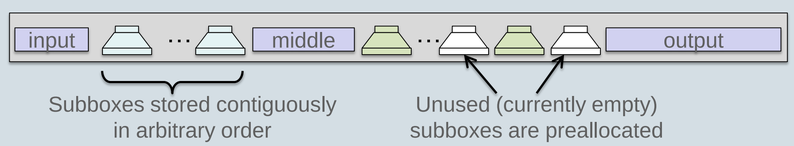




Dynamic Dictionaries [Brodal, Demaine, Iacono, Fineman, Langerrman, Munro 2010]

- Updates are buffered at the nodes + propagated down in batches ➡ updates faster than queries
- Recursive layout of buffers
- Keep pointers between levels to provide efficient searches





References

[1] Bayer, McCreight. *Organization and maintenance of large ordered indexes*. Acta Informatica 1972
[2] Brodal, Demaine, Iacono, Fineman, Langerrman, Munro. *Cache-oblivious dynamic dictionaries with update/query tradeoffs*. SODA 2010
[3] Brodal, Fagerberg. *Lower bounds for external memory dictionaries*. SODA 2003
[4] Brodal, Fagerberg, Jacob. *Cache-oblivious search trees via binary trees of small height*. SODA 2002
[5] Prokop. *Cache-oblivious algorithms*. MIT Master's thesis 1999

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