

Prioritetskøer med Afskæring

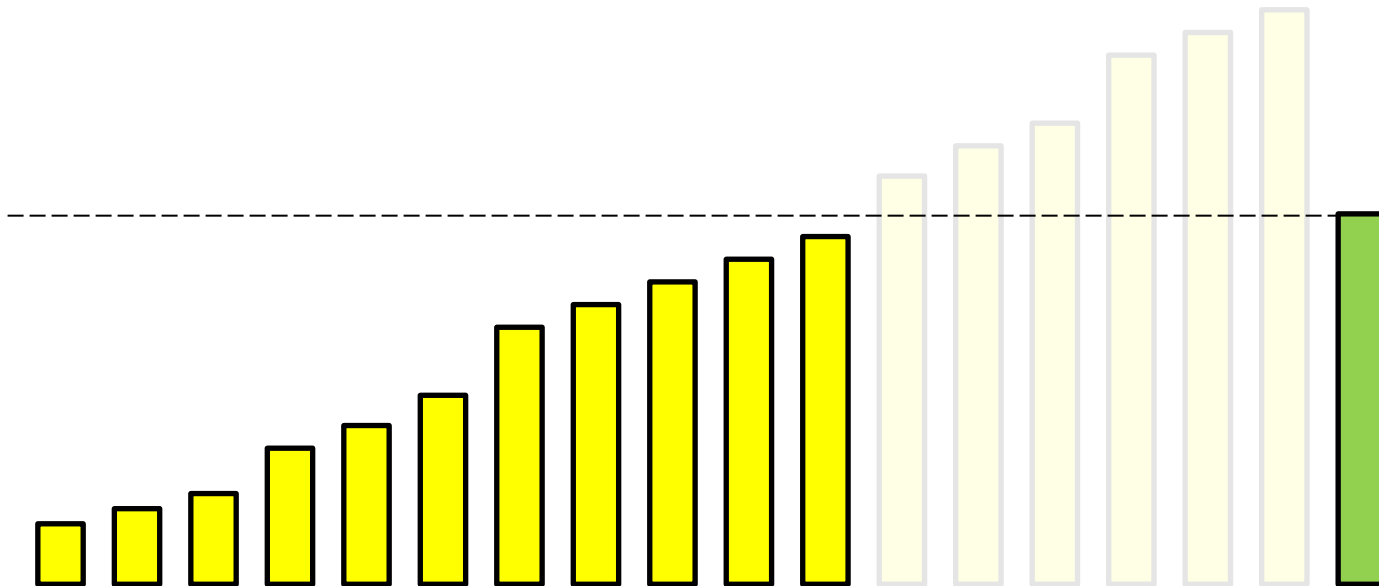
Rajamani Sundar, *Worst-case data structures for the priority queue with attrition*,
Information Processing Letters, 31(2), 69-75, 1989.

Operationer

Create $S := \emptyset$

Insert(x) $S := \{x\} \cup \{y \in S \mid y < x\}$

Deletemin $m := \min(S); S := S \setminus \{m\}; \text{return } m$

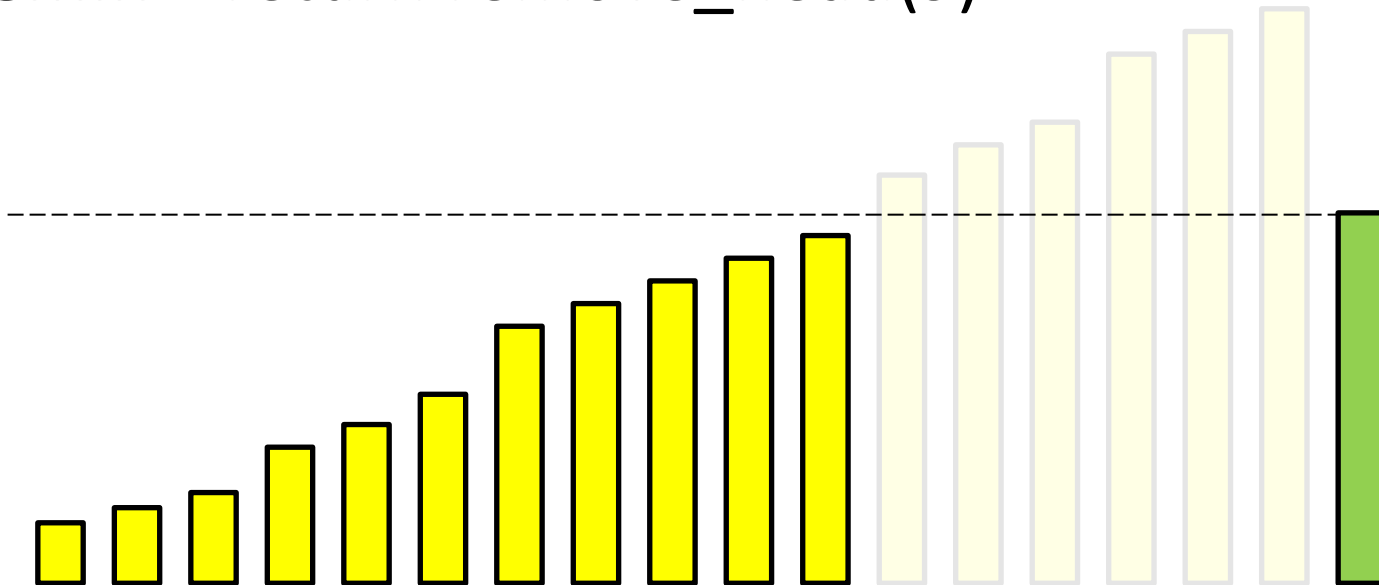


Løsning: Sorteret Liste

Create $S := ()$

Insert(x) while ($|S| > 0$ and $\text{tail}(S) \geq x$) $\text{remove_tail}(S)$
 $\text{insert_tail}(x)$

Deletemin return $\text{remove_head}(S)$



Løsning: Sorteret Liste

Create $S := ()$

Insert(x) while ($|S| > 0$ and $\text{tail}(S) \geq x$) $\text{remove_tail}(S)$
 $\text{insert_tail}(x)$

DeleteMin return $\text{remove_head}(S)$

Sætning

Create, Insert og DeleteMin tager **amortiseret** $O(1)$ tid

Bevis: $\Phi(S) = |S|.$



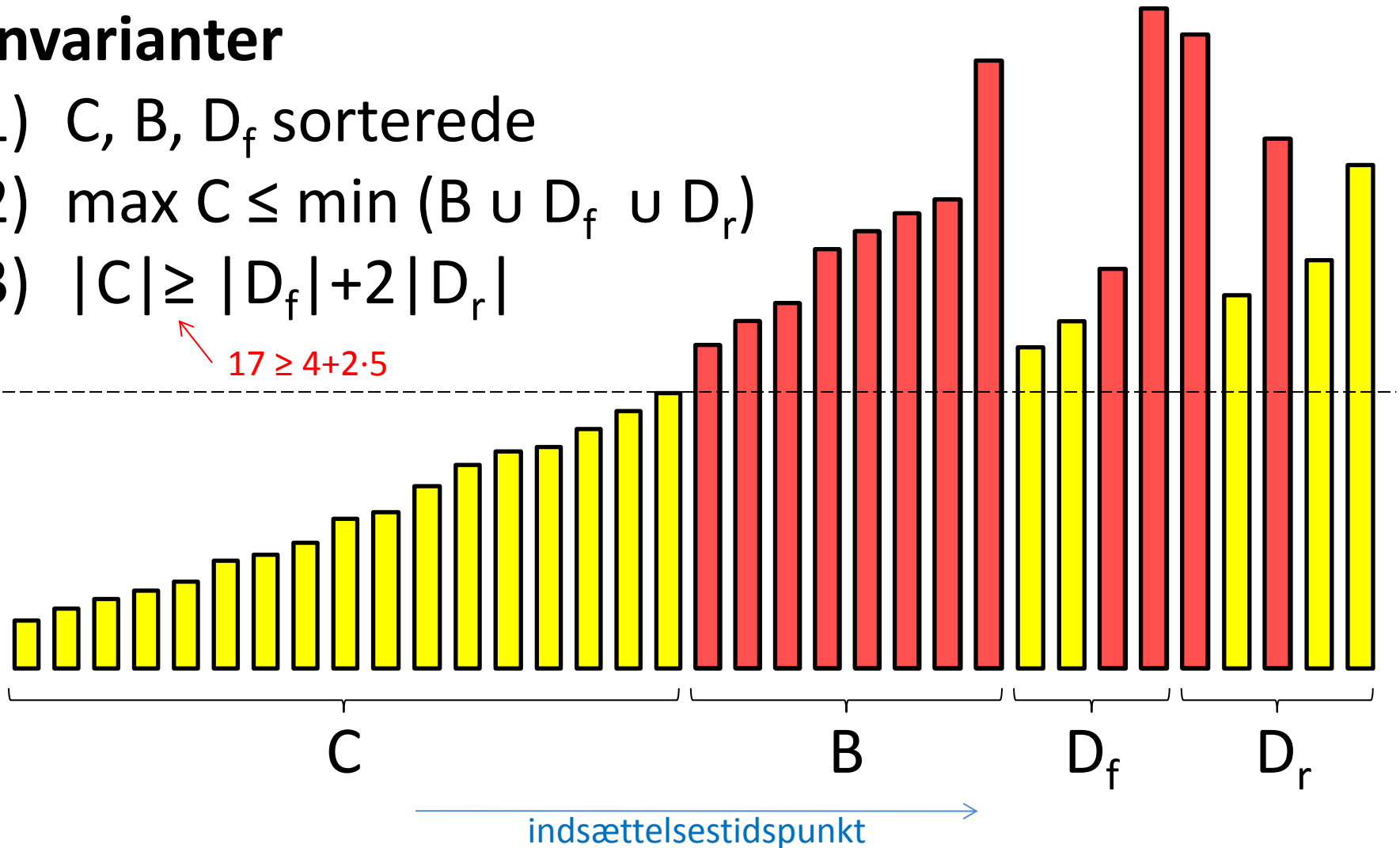
Worst-Case $O(1)$

Løsning: 4 Lister

Invarianter

- 1) C, B, D_f sorterede
- 2) $\max C \leq \min (B \cup D_f \cup D_r)$
- 3) $|C| \geq |D_f| + 2|D_r|$

$17 \geq 4 + 2 \cdot 5$



CREATEPQA \equiv

$C, B, D_f, D_r := (), (), (), ()$

INSERT(x) \equiv

if $C \neq ()$ **and** $\text{first}(C) \geq x$ **then**
{Delete all existing items; add x to C }

① $C, B, D_f, D_r := (x), (), (), ()$

else if $C \neq ()$ **and** $\text{last}(C) \geq x$ **then**

{Empty B, D_f , and D_r ; push back $\text{rest}(C)$ into B ; add x to D_f }

② $C, B, D_f, D_r := (\text{first}(C)), \text{rest}(C), (x), ()$

③ **else** $D_r := D_r \parallel (x)$; **BIAS**; **BIAS**

DELETEMIN \equiv

BIAS;

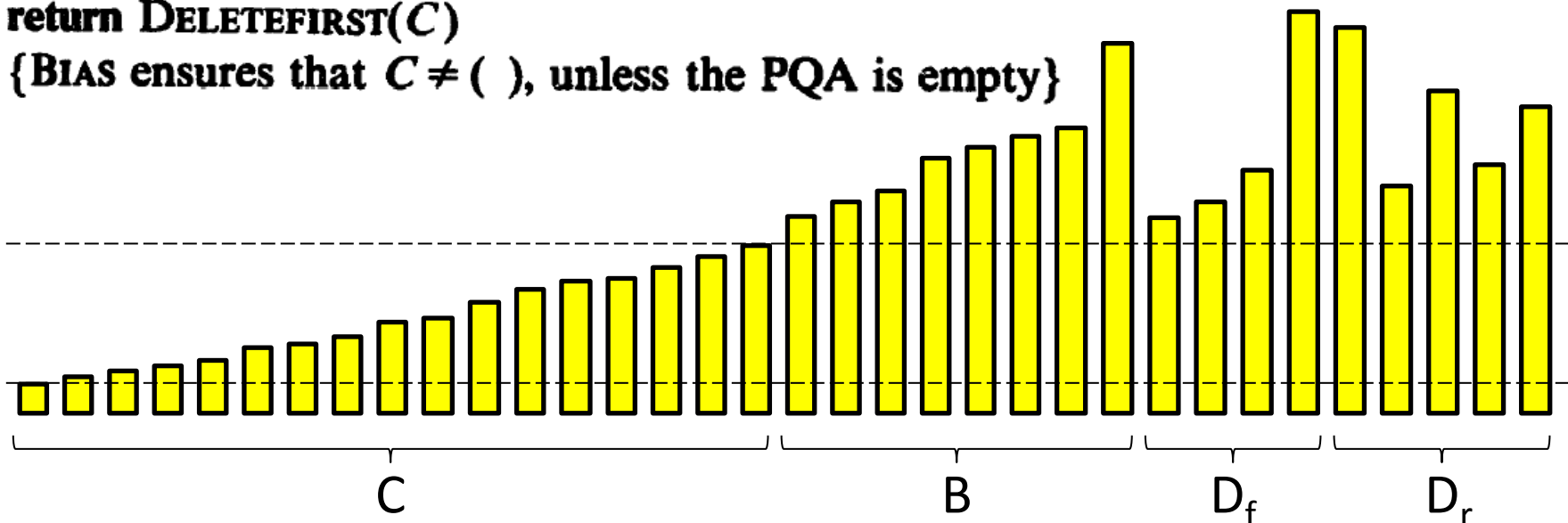
return **DELETEFIRST**(C)

{**BIAS** ensures that $C \neq ()$, unless the PQA is empty}

③

②

①



Invarianter

- 1) C, B, D_f sorterede
- 2) $\max C \leq \min (B \cup D_f \cup D_r)$
- 3) $|C| \geq |D_f| + 2|D_r|$

$\geq "+1"$

BIAS

BIAS \equiv

if $D_r \neq ()$ then

{Clean-up step}

if $D_f \neq ()$ and $\text{last}(D_f) \geq \text{first}(D_r)$ then

A DELETEDLAST(D_f) {decrease $|D_f|$ }

B else PASS(D_f, D_r) {decrease $|D_r|$; increase $|D_f|$ }

else if $D_f \neq ()$ and ($B = ()$ or $\text{first}(B) \geq \text{first}(D_f)$) then

C $D_f, B, C := (), (), C \parallel D_f$ {decrease $|D_f|$; increase $|C|$ }

D else if $B \neq ()$ then PASS(B, C) {increase $|C|$ }

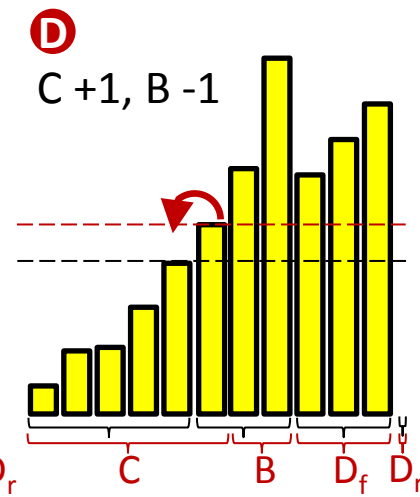
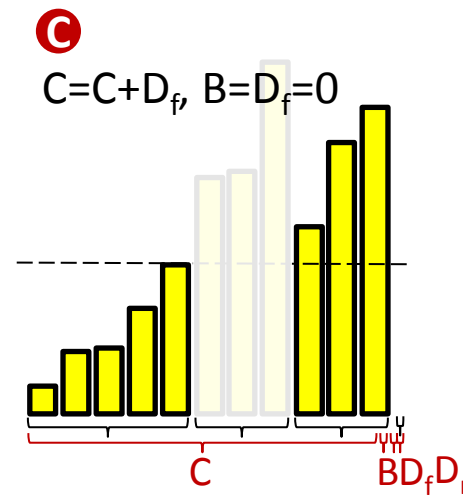
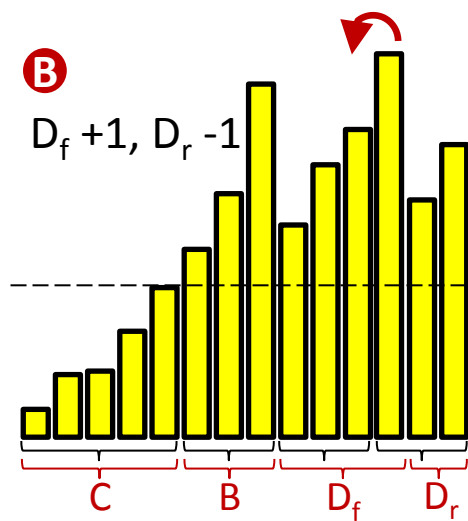
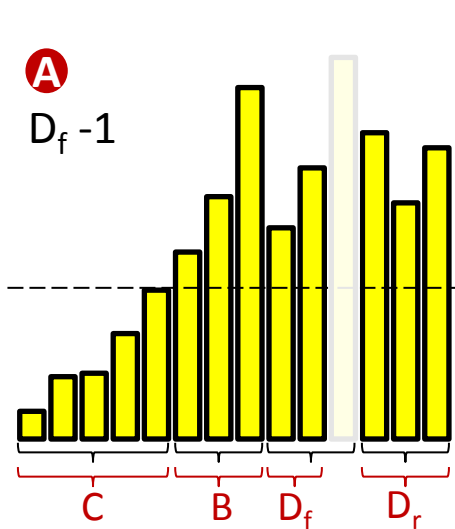
{else $B = D_f = D_r = ()$ }

Invarianter

- 1) C, B, D_f sorterede
- 2) $\max C \leq \min (B \cup D_f \cup D_r)$
- 3) $|C| \geq |D_f| + 2|D_r|$

\geq "+1"

BIAS



Sætning

Create, Insert og DeleteMin tager **worst-case** $O(1)$ tid

Invarianter

- 1) C, B, D_f sorterede
- 2) $\max C \leq \min (B \cup D_f \cup D_r)$
- 3) $|C| \geq |D_f| + 2|D_r|$

