Homework Exercises for Lecture 1

A.1-1 (i) Show that $\lambda_3(n) \geq 5n - 8$.

   (ii) Show that the combinatorial complexity of the lower envelope of n half-lines in the plane is linear.

   (iii) Show that the combinatorial complexity of the lower envelope of n line segments in the plane is linear, if all segments lie in the strip $0 \leqslant x \leqslant 1$ and the x-projection of each segment has length at least c, for some constant fixed constant c.

A.1-2 Consider n points moving on a plane such that the motion of each point p can be described by a polynomial with degree at most s, i.e, $p(t) = (x(t), y(t))$ where $x(t)$ and $y(t)$ are polynomial with degree at most s. Consider the closest pair at initial time. when points start moving, the closest pair may change. First find an instance of n moving points such that the closest pair changes $\Omega(n^2)$ and show that the closest pair changes at most $O(n\lambda_{2s}(n))$.

Hint: First fix a point p and see how many times the closest point to p changes.