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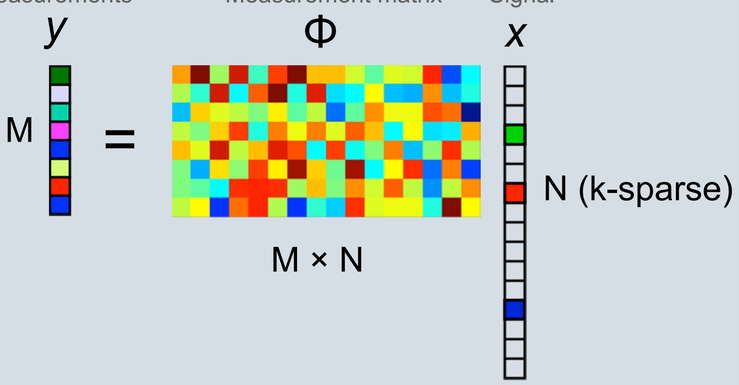
The Constrained Earth Mover Distance Model with Applications to Compressive Sensing

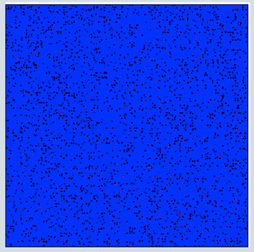
Model-based Compressive Sensing

Compressive sensing

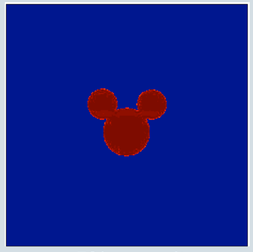
Measurements y = Measurement matrix Φ ($M \times N$) · Signal x (N (k-sparse))



Structured sparsity

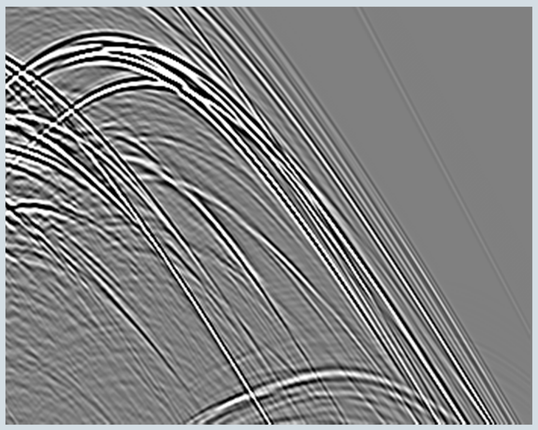

5% sparsity

vs


5% sparsity

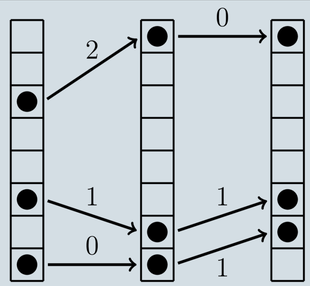
Motivation: Seismic Signals

Part of a shot record from the Sigsbee data set:



The Constrained EMD Model

Idea: restrict how much the support can change from one column to the next.



Definition: EMD

$$\mathrm{EMD}(A,B) = \min_{\pi: A \to B} \sum_{a \in A} |a - \pi(a)|$$

Definition: Support-EMD

$$\mathrm{sEMD}(a,b) = \mathrm{EMD}(\mathrm{supp}(a), \mathrm{supp}(b))$$

Definition: Constrained EMD model

$$A_{k,B} = \{X \in R^{h \times w} : |\mathrm{supp}(x_i)| = k \text{ for } i \in [w], \sum_{i=1}^{w-1} \mathrm{sEMD}(x_i, x_{i+1}) \le B\}$$

Example:

$$X = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 2 \\ 4 & 2 & 0 \end{bmatrix}$$

original signal

$$\hat{X} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 2 \\ 4 & 2 & 0 \end{bmatrix}$$

approximation in model (k = B = 1)

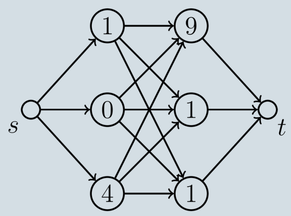
Model Approximation Algorithm

Embed the problem into a **min-cost flow** instance.

Example:

$$X = \begin{bmatrix} 1 & 3 \\ 0 & -1 \\ 2 & 1 \end{bmatrix}$$

$G_{X,k,\lambda} =$



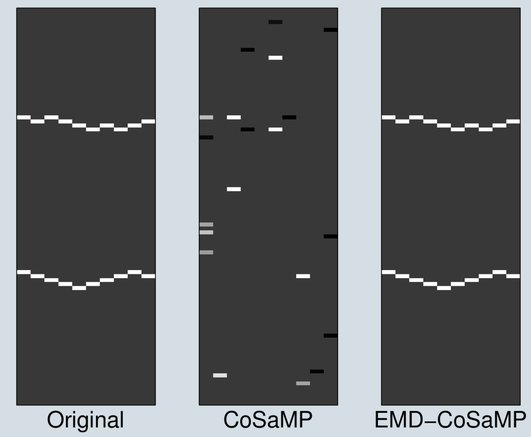
Objective function: $-\|X_S\|_2^2 + \lambda \sum_{i=1}^{w-1} \mathrm{EMD}(s_i, s_{i+1})$

The parameter λ determines the trade-off between EMD and amplitudes. We find a good approximation with a **binary search over λ**.

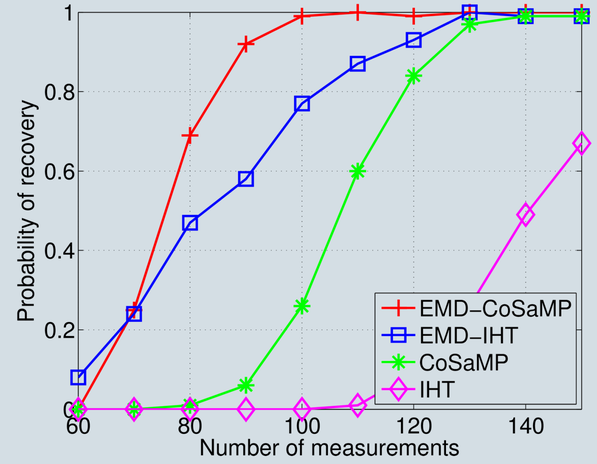
Each min-cost flow instance can be solved in O(k n √n).

Results

Recovery example (parameters: h = 100, w = 10, k = 2, B = 20, m = 80):



Comparison of recovery algorithms (parameters as above, 100 trials):



References

[1] D. Donoho. *Compressed sensing*. IEEE Trans. Inform. Theory 2006.
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