

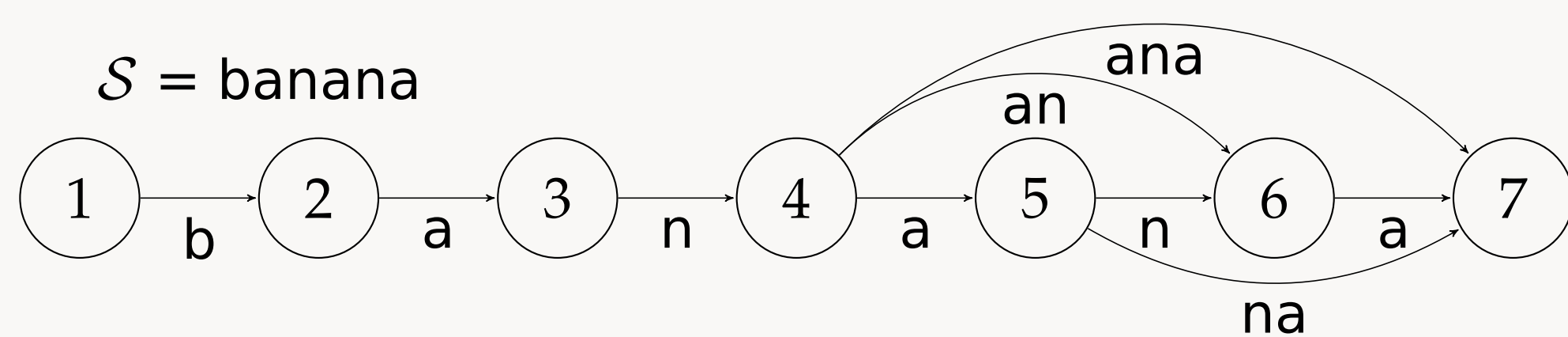
# Bicriteria LZ77 Compression

## The Bicriteria LZ77 Parsing Problem

- ▶ **More than just compression ratio** The advent of massive datasets and high-performing storage systems have reignited the interest towards the design of lossless data compressors which achieve effective compression ratio and very efficient decompression speed.
- ▶ **LZ77** Lempel-Ziv's LZ77 algorithm is the *de facto* choice due to its decompression performance and its algorithmic flexibility, which allow to trade decompression speed for compression ratio.
- ▶ **Picking between different trade-offs** Each existing implementation offers a *single* trade-off between space occupancy and decompression speed, so software engineers have to content themselves by picking the one which comes closer to the requirements of the application in their hands.
- ▶ **The Bicriteria LZ77 Parsing Problem** Find a parsing which minimize the consumption of one resource (decompression time, compressed size) given a bound on the consumption of the other one.

## Modeling as a WCSPP

- ▶ The set of LZ77 parsings of a string  $\mathcal{S}$  of length  $n$  may be expressed as the source-destination paths over a graph  $\mathcal{G}(\mathcal{S})$  with  $O(n^2)$  edges, such that:
  - there is a vertex  $v_i$  for each character  $\mathcal{S}[i]$ ;
  - there is an edge  $(v_i, v_j)$  for each substring  $\mathcal{S}[i, j - 1]$  in the dictionary.



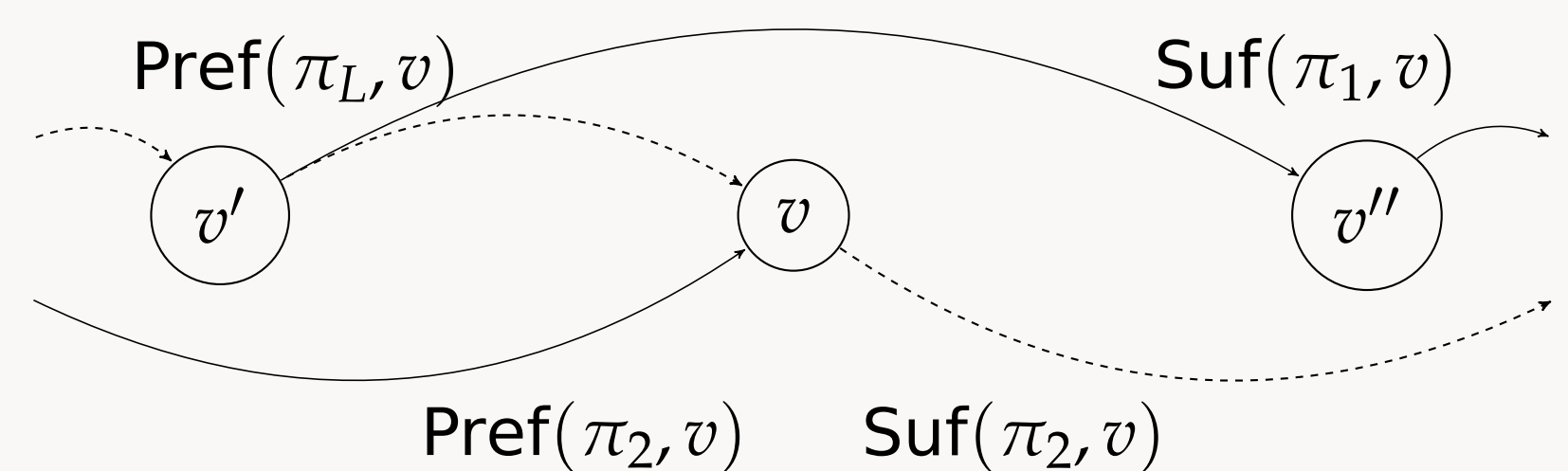
- ▶ each edge, which correspond to a phrase, is weighted with a *time* and *space* weight.
  - the space weight is the codeword length in bits, while its time weight is given by an experimental, scan-based time model.
- ▶ The Bicriteria LZ77 Parsing Problem is thus reduced to a **Weight-Constrained Shortest Path Problem** over  $\mathcal{G}(\mathcal{S})$ .

## Solving the WCSP on $\mathcal{G}(\mathcal{S})$

- ▶ General-purpose WCSPP resolution algorithms are not appropriate in this context.
  - the graph may be very huge: the number of edges of the graph induced by a one-gigabyte file can be up to  $2^{32 \cdot 2} = 2^{64}$  edges, which make storing it unfeasible.
  - state-of-the-art algorithms for WCSPP, when applied to the Bicriteria LZ77 Parsing problem, have a complexity of at least  $\Omega(n^2)$ , which is unacceptable in practice.
- ▶ Our algorithm exploits some peculiar structural properties of  $\mathcal{G}(\mathcal{S})$  to achieve  $O(n \log^2 n)$  time and  $O(n)$  auxiliary space complexity.
- ▶ The algorithm is an **additive**  $(O(\log n), O(\log n))$ -approximation algorithm.
  - Assuming that the optimal solution has compressed size  $s$  and the decompression time bound is  $T$ , the algorithm finds a solution with compressed space and decompression time bounded by  $s + O(\log n)$  and  $T + O(\log n)$ .
  - Very close to the optimum, even on small files.

## Our solution

- ▶ Our solution can be decomposed in four steps.
- ▶ **Pruning** Under some broad assumptions about the encoding functions and the memory hierarchy, the number of edges may be reduced from  $O(n^2)$  to just  $O(n \log n)$  in an implicit fashion.
- ▶ **Forward Star Generation** Each edge is dynamically generated when needed in  $O(1)$  amortized time, in order to achieve  $O(n)$  space complexity.
- ▶ **Lagrangian Relaxation** We solve the Lagrangian Dual relaxation of the WCSPP in  $O(n \log^2 n)$  time through the Cutting Plane algorithm. This phase yields a lower and upper-bound on the cost of the optimal solution, plus a pair of paths  $(\pi_L, \pi_R)$  which constitute an *optimal basis* of the dual problem.
- ▶ **Approximate Gap-closing** We obtain an additive  $(O(\log n), O(\log n))$ -approximation by combining together the paths  $\pi_L$  and  $\pi_R$  in  $O(n)$  time and space. The resulting path is composed by a prefix of  $\pi_L$  and a suffix of  $\pi_R$  starting from a carefully-picked vertex  $v$ , plus a *swapping bridge* connecting the two sub-paths.



## Experimental Results (DBLP, 1GB)

Parsing	Compressed size (MB)	Decompression time (seconds)
BC-ZIP - 1	129.8	2.95
BC-ZIP - 0.8	131.4	2.77
BC-ZIP - 0.6	134.6	2.56
BC-ZIP - 0.4	139.3	2.32
<b>BC-ZIP - 0.2</b>	<b>148.5</b>	<b>1.96</b>
Snappy	323.4	2.13
LZ4	214.7	1.98
zlib	190.5	11.65
bzip2	121.4	48.98

Experimental results show that our approach allows to effectively control the time-space trade-off in a practical yet principled manner. Moreover, it leads to parsings which are faster to decode and more space-succinct than those generated by highly tuned and engineered parsing heuristics, like those of Google Snappy and LZ4

## References

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