MADALGO
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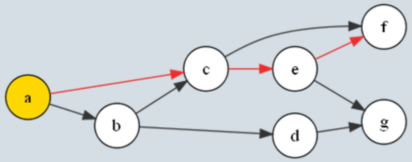
Improved Analysis of the Average-Case Behavior of Classic Single-Source Shortest Path Approaches

Problem and Algorithms

Single-Source Shortest-Paths



- Input is a graph with *n* vertices and *m* arcs
- Find the shortest path from source vertex *s* to all other vertices
- We study the lower bound of the runtime on graphs with independent real random edge weights uniformly chosen from the interval [0,1]

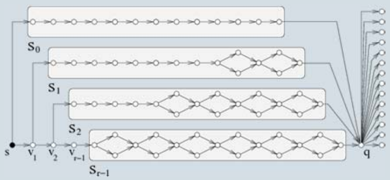
List class algorithms
(Maintain one or more queues)

- Bellman-Ford algorithm
- Pallottino's algorithm

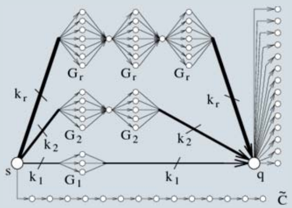
Algorithms with Approximate Priority Queues
(Maintain arrays of buckets that contain an interval of tentative distances)

- Approximate Bucket Implementation of Dijkstra's algorithm (ABI-Dijkstra)
- Δ-Stepping algorithm

Previous Worst-Case Constructions



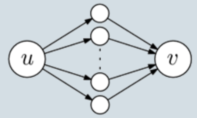
- Causes Bellman-Ford and Pallottino's algorithm to run in $\Omega(n^{4/3-\varepsilon})$



- Causes ABI-Dijkstra to run in $\Omega(n \log n / \log\log n)$
- Causes Δ-Stepping to run in $\Omega(n \log n / (\log\log n)^{1/2})$

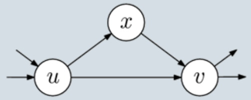
Building Blocks

(u,v,k)-Gadgets



- k disjoint paths of length 2 between vertices u and v
- The higher k, the lower the expected shortest path weight

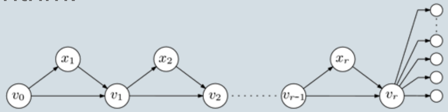
Triangle subgraph



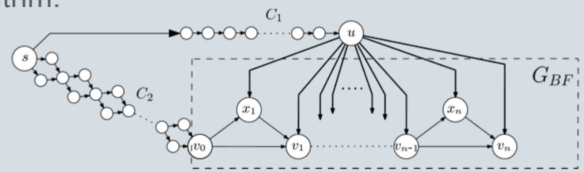
- The path from u to v via x is smaller than the direct connection with probability 1/6

New Worst-Case Instances

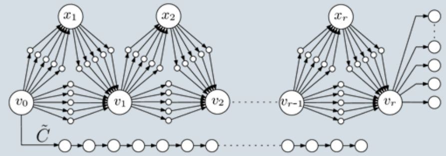
For the Bellman-Ford algorithm:



For Pallottino's algorithm:



For algorithms with approximate priority queues:



Algorithm	Previous Lower Bound	New Lower Bound	Upper Bound
Bellman-Ford algorithm	$\Omega(n^{4/3-\varepsilon})$	$\Omega(n^2)$	$O(n^2)$
Pallottino's algorithm	$\Omega(n^{4/3-\varepsilon})$	$\Omega(n^2)$	$O(n^3)$
ABI-Dijkstra	$\Omega(n \log n / \log\log n)$	$\Omega(n^{1.2-\varepsilon})$	$O(n^2)$
Δ-Stepping	$\Omega(n \log n / (\log\log n)^{1/2})$	$\Omega(n^{1.1-\varepsilon})$	$O(n^2)$

MADALGO – Center for Massive Data Algorithmics, a Center of the Danish National Research Foundation