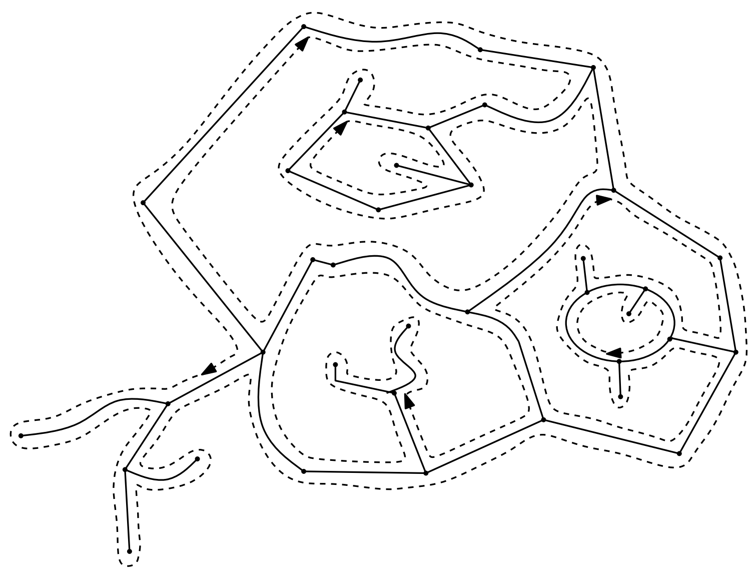
Extension of Planar Graph Drawings : Maximum Fan-crossing Free Graphs

Otfried Cheong[1], Sariel Har-Peled[2], Hyosil Kim[3], Hyuna Kim[1]

[1]KAIST, Daejeon, Korea
[2]UIUC, Urbana, IL, USA
[3]POSTECH, Pohang, Korea

Background : The Maximum Number of Edges in a Planar Graph is $3v - 6$

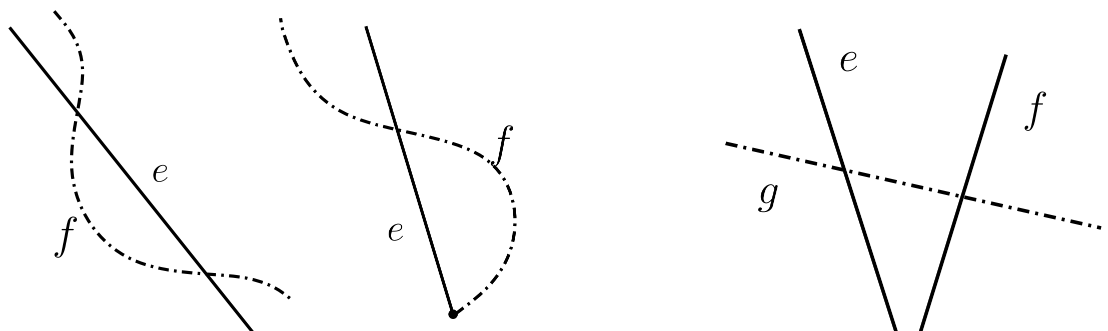
- for a planar graph $G = (V, E)$ with $|V| = v$.
- Triangulation. $\equiv$ # edges reaches the maximum
  - $3f \leq \sum_{F:faces}(\text{ \# edges incident to } F) \leq 2e$



  - Euler's Fomula ($v - e + f = 2$).
  - where v, e, f: # vertices, edges, and faces.
  - $v - e + f = 2 \leq v - e + \frac{2}{3}e = v - \frac{1}{3}e$.

Definitions : Fan-crossing Free Graphs

- We assume that drawings do not allow



(a) (b) Fan-crossing

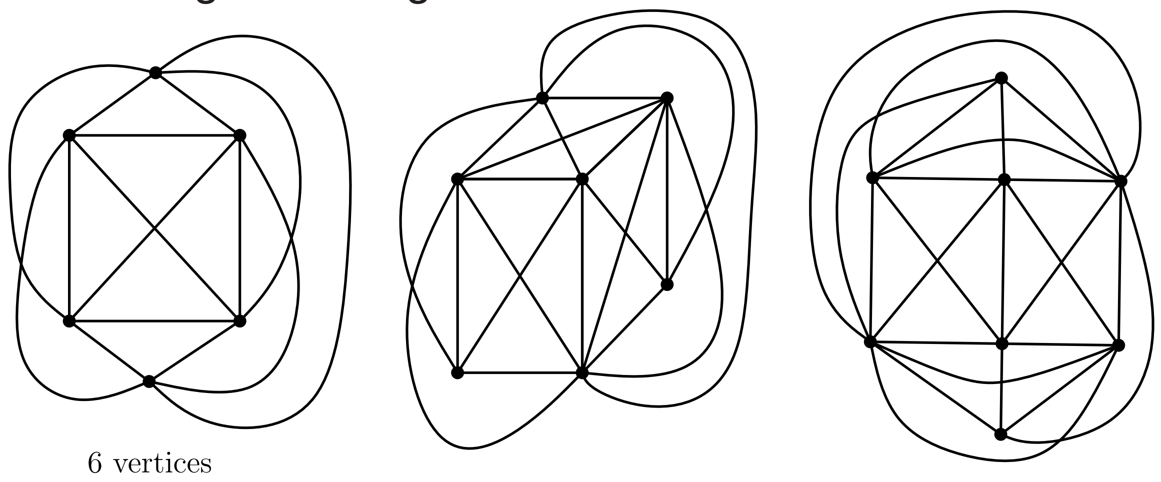
- A Maximal Fan-crossing Free Graph G
  - G is simple. (No parallel edges or loops.)
  - No more edge can be added to G for a given drawing.
- A Maximum Fan-crossing Free Graph G
  - G is simple.
  - G achieves the maximum number of edges for a given number of vertices.

Conjectures : Maximum Fan-crossing Free Graphs

- $e = 4v - 8$ for non-straight line drawings
- $e = 4v - 9$ for straight line drawings
  - where e, v : # edges, vertices
- 2-connected
  - connected and a removal of a vertex does not affect to its connectivity.
- It can be constructed by tiling K_4's.
  - It contains an underlying triangulation of vertices.
  - Edges can be added to every two adjacent triangles

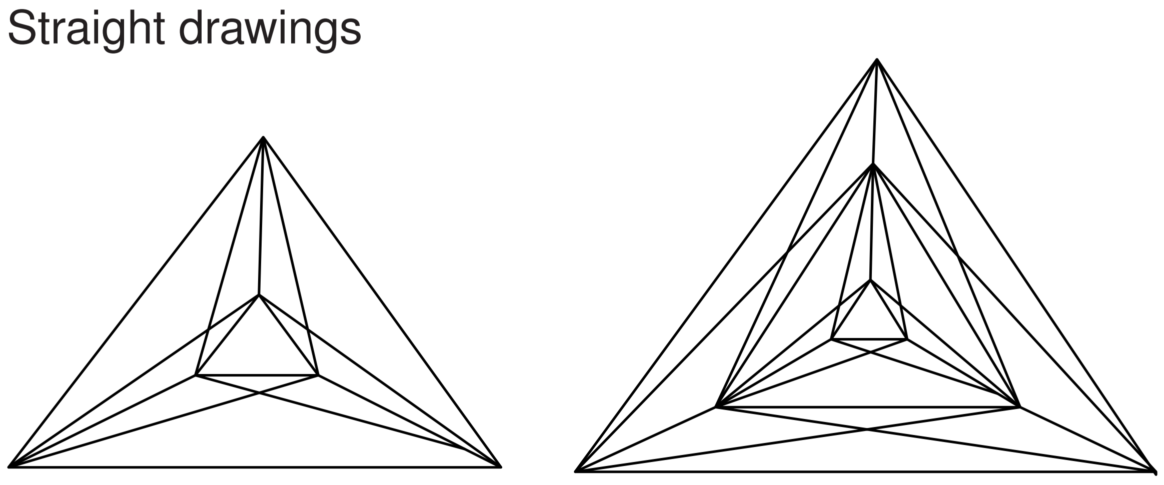
Proofs - A Lower Bound (Work in Progress) : Maximum Fan-crossing Free Graphs

- Non-straight drawings



6 vertices, 15 edges, the complete graph — 7 vertices, 20 edges — 8 vertices, 24 edges

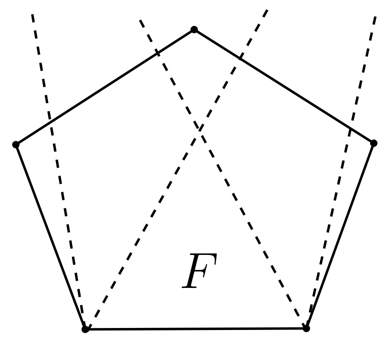
- Straight drawings



6 vertices, 15 edges — 9 vertices, 27 edges

Proofs - An Upper Bound (Work in Progress) : Maximum Fan-crossing Free Graphs

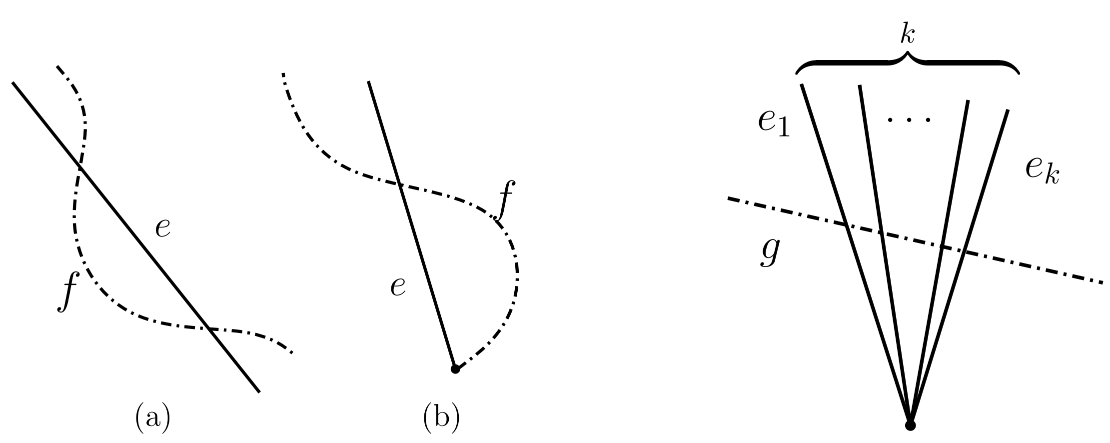
- If a face is bounded by some cycle, fan-crossing graphs
  - can be obtained by adding a restricted #edges to each face,
  - proved by induction.



- If a face is not bounded by a cycle,
  - take a closed walk along the boundary of a faces.
  - handle holes (if there is no closed walk).

Further Study (Work in Progress) : Maximum k-bound Free Graphs

- We assume that drawings do not allow



(a) (b) k-bound

Computer Science Department - KAIST - Daejeon, Korea

Mail: hyunak@kaist.ac.kr